Princeton Physics Preliminary Exam Formulas
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1 Math

Solution to Laplace’s equation in cylindrical coordinates
\[ V(r, \theta) = A + B \ln r + \sum_{k=1}^{\infty} \left( C_k r^k + D_k r^{-k} \right) (E_k \cos k\theta + F_k \sin k\theta) \]

Solution to Laplace’s equation in spherical coordinates
\[ V(r, \theta) = \sum_{l=0}^{\infty} \left( A_l r^l + B_l r^{-l-1} \right) \Phi_l(\cos \theta), \Phi(\phi) = \]

First 3 Legendre polynomials
\[ P_0(x) = 1, P_1(x) = x, P_2(x) = \frac{3x^2 - 1}{2} \]

Law of cosines
\[ c^2 = a^2 + b^2 - 2ab \cos \theta \]

2 Mechanics

2.1 Lagrangian Mechanics

Lagrangian definition
\[ \mathcal{L} = T - V \]

Euler-Lagrange equation
\[ \frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{q}} = \frac{\partial \mathcal{L}}{\partial q} \]

Conserved quantities of motion from a Lagrangian
If \( \frac{\partial \mathcal{L}}{\partial \dot{q}} = 0 \), then \( p = \frac{\partial \mathcal{L}}{\partial \dot{q}} \) is conserved. \( \left( \frac{d}{dt} = 0 \right) \)

Hamiltonian in terms of Lagrangian
\[ \mathcal{H}(p, q, t) = \sum_{l=0}^{\infty} p_l q_l - \mathcal{L}(q, \dot{q}, t) \]

Hamilton’s equations
\[ \frac{dq}{dt} = \frac{\partial \mathcal{H}}{\partial p}, \quad \frac{dp}{dt} = -\frac{\partial \mathcal{H}}{\partial q} \]

2.2 Harmonic oscillators

Undamped frequency of a harmonic oscillator
\[ \omega_0 = \sqrt{\frac{k}{m}} \]

Quality factor definition
\[ Q = \sqrt{\frac{mk}{\gamma}} \]

Form of a damped harmonic oscillator
\[ \ddot{x} + \frac{\omega_0^2}{Q} \dot{x} + \omega_0^2 x = 0 \]

2.3 Central force problems

Lagrangian for a central force orbit
\[ \mathcal{L} = \frac{1}{2} m \dot{r}^2 + \frac{1}{2} m (\dot{r} \dot{\phi})^2 - V(r) \]

Effective potential for central force problems
\[ V_{eff} = V(r) + \frac{\ell^2}{2mr^2}. \text{ Derive from central force Lagrangian.} \]

In a \( -k/r \) potential, what do the orbits look like?
\[ r(\phi) = \frac{P}{1 + \epsilon \cos (\phi - \phi_0)} \text{ where } P = \ell/mk, \text{ and } \epsilon = \pm \sqrt{1 + 2E \ell^2 / mk} \]

Find the differential cross section
\[ \frac{d\sigma}{d\Omega} = \frac{k}{\sin(\phi)} \frac{dp}{d\phi} \]

On an ellipse, what is the periapsis?
The distance of closest approach
2.4 Rigid body motion

Torque about center of mass
\[ \tau = \frac{dL}{dt} _{\text{about \ CM}} \]

Rotational angular momentum
\[ \vec{L} = I \vec{\omega} \]

Parallel axis theorem
\[ I = I_{CM} + MR^2 \]

Orbital angular momentum
\[ L_{CM} = M(\vec{v}_{CM} \times \vec{v}_{CM}) \]

Rotational kinetic energy
\[ K = \frac{1}{2} I \omega^2 \]

Velocity of a point on a rotating body away from the center of mass
\[ \vec{v}_{\text{point}} = \vec{v}_{CM} + \omega \times r \]

No slip condition
\[ 0 = \vec{v}_{\text{cm}} + \omega \times r \]

Rotating reference frame derivative shift for a vector
\[ \left( \frac{dA}{dt} \right)_{\text{inertial}} = \left( \frac{dA}{dt} \right)_{\text{non-inertial}} + \Omega \times A \]

Using the above identity, we can derive Euler’s identities. We use \( \vec{A} = \vec{L} \), the spin angular momentum. In an inertial reference frame \( dL/dt = 0 \), so we are left with the following. I will expand it
\[ I_1 \omega_1 + (I_3 - I_2) \omega_2 \omega_3 = 0 \]  

Euler equations for rotating bodies
\[ I_1 \omega_1 + (I_3 - I_2) \omega_2 \omega_3 = 0 \text{ permute } ijk \]

Wave equation on a string in horizontal tension.
\[ \partial^2 v = c^2 \partial^2 y \text{ where } c^2 = T_R / \mu. \]

How do you derive the fictitious forces? Use \( \left( \frac{dA}{dt} \right)_{\text{inertial}} = \left( \frac{dA}{dt} \right)_{\text{non-inertial}} + \Omega \times A \) with \( \vec{v} = d\vec{x}/dt \) to write the Lagrangian for a free particle in a rotating frame. Compute the Euler Lagrange equations for the equations of motion. There are no real forces, only fictitious forces. The ABC, BAC, CAB dot cross product identities are used here.

Conservation of energy
\[ P + \frac{1}{2} \rho v^2 + \rho gh = \text{const.} \]

Navier stokes
\[ \rho \partial \vec{v} / \partial t + \nabla \cdot (\rho \vec{v} \vec{v}) = - \nabla P + \eta \nabla^2 \vec{v} + \rho F_{\text{ext}}. \]

3 E&M

Reflection and transmission coefficients
\[ t = \frac{2n_1}{n_1 + n_2}, \quad r = \frac{n_2 - n_1}{n_1 + n_2} \]

Maxwell Stress
\[ T_{ij} = \varepsilon [E_i E_j + c^2 B_i B_j - \frac{1}{2} (E^2 - c^2 B^2) \delta_{ij}] \]
Capacitance definition and energy

\[ Q = CV \quad E = \frac{1}{2}QV = \frac{1}{2}CV^2 \]

Capacitance of parallel plates

\[ C = \varepsilon \frac{A}{d} \]

B field from a wire

\[ B = \frac{\mu_0 I}{2\pi r} \]

B field inside a solenoid

\[ B = \mu NI \]

D in terms of E

\[ D = \varepsilon E = \varepsilon_0 E + P \]

H in terms of B

\[ H = B/\mu = B/\mu_0 - M \]

Discontinuity condition for electric field

\[ \Delta E_\perp = \sigma B/\varepsilon \quad \Delta E_\parallel = 0 \]

Discontinuity condition for magnetic field

\[ \Delta B_\parallel = \mu \hat{z} \quad \Delta B_\perp = 0 \]

Perpendicular means perpendicular to the plane of the surface. Both of these conditions are for the displacement fields \( D, H \) instead of for \( E, B \) really.

Superconducting boundary condition

\[ B = 0 \] on the surface

3.1 Method of images conditions

Method of images: charge condition

Opposite charge and position

Method of images: dipole charge on conducting boundary

Flip along the axis perpendicular to the plane

Method of images: magnetic dipole condition

Mirror the dipole across the boundary

Point charge and sphere

\[ q' = -qR/a \] and \[ b = R^2/a \]

Electric and magnetic potential definitions

\[ \vec{E} = -\nabla V - \frac{\partial \vec{A}}{\partial t}, \quad \vec{B} = \nabla \times \vec{A} \]

Poynting vector and E&B field momentum

\[ \vec{S} = \frac{1}{\mu_0} (\vec{E} \times \vec{B}) = \vec{g} = \varepsilon E \times B = D \times B \]

Energy density in field

\[ U/V = \frac{1}{2} \left[ \varepsilon E^2 + \frac{1}{\mu_0} B^2 \right] \]

Brute force magnetic potential (A) integral

\[ A = \frac{\mu_0}{4\pi} \int \frac{j(\vec{r})}{|r'^2|} dV' \]

Brute force electric potential (V) integral

\[ V = \frac{1}{4\pi\varepsilon} \int \frac{\partial \phi(\vec{r})}{\partial t} dV' \]

Brute force magnetic field (B) integral

\[ B = \frac{\mu_0}{4\pi} \int \frac{\partial \vec{A}}{\partial t} \hat{r} dV' \]

Brute force electric field (E) integral

\[ E = \frac{1}{4\pi\varepsilon} \int \frac{\partial \phi}{\partial t} \hat{r} dV' \]

Magnetic moment integral

\[ m = \frac{1}{2} \int j \times r \times dV' \]

Torque on a dipole

\[ \vec{\tau} = \vec{m} \times B_{ext} \]

Energy of a dipole in a magnetic field

\[ U = -m \cdot B_{ext}. \] Remember minus because dipoles want to align with B field

Force on a dipole

\[ \vec{F} = \nabla (m \cdot B_{ext}) \]
Angular momentum of E&B field
\[ \vec{l} = \vec{r} \times \vec{g} = \vec{r} \times \vec{\varepsilon} (E \times B) \]

Magnetic field from dipole
\[ \vec{B}_{\text{dip}} = \frac{\mu_0}{4\pi} \frac{3(\vec{m} \cdot \hat{r}) \hat{r} - \vec{m}}{r^3} \]

Magnetic potential from a dipole
\[ \vec{A} = \frac{\mu_0 \vec{m} \times \hat{r}}{4\pi r^2} \]

Power and force of an accelerating charge
\[ P = \frac{\mu_0}{6\pi c q^2} a^2, \quad F = \frac{\mu_0}{6\pi c} q \dot{a} \]

Current in terms of electric field
\[ J = \sigma \vec{E} \]

Polarization in terms of displacement
\[ P = N q \vec{r} \]

TE vs TM vs TEM
- TE: \( E_z = 0 \)
- TM: \( B_z = 0 \)
- TEM: \( B_z = E_z = 0 \)

Let's derive a dispersion relation in a conductive medium.
\[ \nabla \times \vec{B} = \mu \vec{J} + \frac{1}{c^2} \frac{\partial \vec{E}}{\partial t} \quad (10) \]
\[ \nabla \times \nabla \times \vec{B} = \sigma \mu \nabla \times \vec{E} + \frac{1}{c^2} \frac{\partial}{\partial t} (\nabla \times \vec{E}) \quad (11) \]
\[ -\nabla^2 \vec{B} = -\sigma \mu \frac{\partial \vec{B}}{\partial t} - \frac{1}{c^2} \frac{\partial^2 \vec{B}}{\partial t^2} \quad (12) \]
\[ k^2 \vec{B} = i \omega \sigma \mu \vec{B} + \frac{1}{c^2} \omega^2 \vec{B} \quad (13) \]
\[ c^2 k^2 = i \omega \mu \sigma c^2 + \omega^2 \quad (14) \]

4 Quantum
4.1 Mathy things

Unitary definition
\[ f(-a) = f^\dagger(a) \text{ i.e. inverse = conjugate transpose} \]

Commutator \([A, B] = AB - BA\)

Expectation value of an operator
\[ <A> = \sum_n c_n^* A_n c_n \]

Orthornormality and eigenvalues
\[ <\psi_n|\psi_m> = \delta_{nm}, \quad \hat{A} \psi_n = a_n \psi_n, \quad \psi = \sum_n c_n \psi_n, \quad c_n = <\psi_n|\psi> \quad P_n = |c_n|^2 \]

Hermitian operator definition and important property \( H = H^\dagger, \quad <v|H|w> = <H^\dagger|v|w> \)

Inner product (vectors and functions)
\[ <v|w> = \sum_i v^*_i w_i = v^\dagger w, \quad <f|g> = \int f(x) g(x) dx \]

Energy and momentum in QM
\[ E = \hbar f = \hbar \omega, \quad p = \hbar k = \hbar / \lambda \]

Ehrenfest Theorem
\[ \frac{d<A>}{dt} = <\frac{\partial A}{\partial t}> + \frac{i}{\hbar} <[H, A]> \]

Generalized uncertainty principle
\[ \sigma_A^2 \sigma_B^2 \geq \left( \frac{1}{i} <[A, B]> \right)^2 \]

Density operator and matrix elements
\[ \hat{\rho} = \sum p_k |\psi_k> <\psi_k|, \quad A_{ij} = <\psi_i|\hat{A}|\psi_j> \]

4.1.1 Fundamentals related to 1D SE

Energy operator \( \hat{H} \)
\[ \hat{H} = \frac{p^2}{2m} + V(x, t) = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V(x, t) \]

Schroedinger equation
\[ i\hbar \frac{\partial \psi}{\partial t} = \hat{H} \psi \]

Position and momentum operators
\[ \hat{x} = x, \quad \hat{p} = \hbar \frac{\partial}{\partial x} \]

Time Independent Schrodinger equation
\[ \hat{H} \psi = E \psi \]
4.2 1D problems

Infinite square well

\[ q^2 \frac{\sin (n \pi x / L)}{L} \]

Infinite square well Energy levels

\[ E_n = \frac{n^2 \pi^2 \hbar^2}{2mL^2} \]

Delta function discontinuity condition. \( V(x) = a \delta(x) \)

Integrate SE over each side of the \( \delta \) function and show \( \Delta \frac{\partial}{\partial x} \psi(x) |_{x_0} = \frac{2ma}{\hbar} \psi(x_0) \)

1D Time independent wave function construction.

\[ \psi_n(x,t) = \psi_n(x) e^{-iE_n t / \hbar} \]

\[ H\psi_n = E_n \psi_n \]

\[ c_n = \langle \psi_n | \psi(x,0) \rangle \]

4.2.1 Harmonic oscillator

Harmonic oscillator raising and lowering operators and normalization.

\[ H = \frac{p^2}{2m} + \frac{1}{2} m \omega^2 x^2 \]

\[ X = x \sqrt{\frac{m}{\hbar}} \quad P = p \sqrt{\frac{m}{\hbar}} \]

\[ a_- = \frac{X - iP}{\sqrt{2}} \quad a_+ = \frac{X + iP}{\sqrt{2}} \]

Determine ground state of harmonic oscillator

\[ a_- \psi_0 = 0 = \left( \frac{X + iP}{\sqrt{2}} \right) \psi_0 = 0 \]

Energies of 3D SHO

\[ E_n = \hbar \omega (f/2 + n) \]

Raising and lowering operations on wavefunctions

\[ a_- \psi_n = \sqrt{n} \psi_{n-1} \quad a_+ \psi_n = \sqrt{n+1} \psi_{n+1} \]

Ground state of SHO

\[ \psi = N_0 e^{-x^2 / 4\hbar} \]

4.3 2 particle systems

Wavefunction and energies of 2 non-interacting distinguishable particles

\[ \psi(r_1, r_2) = \psi_a(r_1) \psi_b(r_2) \quad E = E_a + E_b \]

Wavefunction and energies of 2 indistinguishable identical particles

\[ \psi \pm(r_1, r_2) = A \left[ \psi_a(r_1) \psi_b(r_2) \pm \psi_b(r_1) \psi_a(r_2) \right] \quad E_a + E_b \]

4.4 3D problems and hydrogen atom

Bohr condition

\[ l = \hbar n \] angular momentum is quantized

Spherical harmonic \( Y_{0,0} \)

\[ Y_{0,0} = 1 / \sqrt{4\pi} \]

Spherical harmonic \( Y_{1,0} \)

\[ Y_{0,0} = N_{10} \cos \theta \]

Spherical harmonic \( Y_{1,1} \)

\[ Y_{0,0} = N_{11} \sin \theta e^{i\phi} \]

Ground state wavefunction for hydrogen

\[ e^{-r/a_B} / \sqrt{4\pi a_B} \]

Bohr Radius

\[ a_B = \frac{4 \pi \epsilon_0 m_e^2 k^2}{\hbar^2} \] where \( n=1 \)

Hydrogen energy levels

\[ E = \frac{1}{2} U = \frac{m_e}{2 m} \left( \frac{a}{2 \pi \epsilon_0} \right)^2 \]

Use Virial theorem.
4.5 Orbital stuff, angular momentum, and spin

Orbital angular momentum EV equations

\[ L^2 \left| \ell m \right> = \ell^2 \left( \ell + 1 \right) \left| \ell m \right> \quad \text{and} \quad L_z \left| \ell m \right> = \ell m \left| \ell m \right> \]

Commutators of angular momentum operators

\[ [L_x, L_y] = \imath \hbar L_z \text{ and permutations therefore} \]

Raising and lowering angular momentum operators

\[ L_+ = L_x \pm \imath L_y \]

Eigenvalues equation of raising and lowering operators

\[ L_{\pm} \left| \ell m \right> = \hbar \sqrt{\ell(\ell + 1) - m(m \pm 1)} \left| \ell m \pm 1 \right> \]

Figure out signs of signs by using \( L_{\pm} \left| \ell \ell \right> = 0 \) and \( L_{-} \left| \ell - \ell \right> = 0 \)

How do the spin and orbital angular momentum EV equations differ

They don’t. Just use \( S = L \) and, \( s = l \)

Pauli spin matrix

\[ \begin{bmatrix} z & x - iy \\ x + iy & -z \end{bmatrix} \]

1st column positive, 2nd column negative

What are Pauli spin matrices used for?

Measuring the wavefunction \( \chi \) in the spin directions.

\[ c_{m,n} = \sigma_x \chi \]

Eigenvalues of \( E \) for \( H = \sigma_1 \cdot \sigma_2 \)

Singlet \( E = \frac{\mathbb{I}}{2} \)

Triplet \( E = -\frac{\mathbb{I}}{2} \)

4.6 Perturbation theory

Let’s derive the 1st order corrections. First, expand EVERYTHING to an order in \( \lambda \)

\[ \left( \hat{H}^0 + \lambda \hat{H}^1 \right) \left( \psi_n^0 + \lambda \psi_n^1 + \ldots \right) = \left( E_n^0 + \lambda E_n^1 + \ldots \right) \left( \psi_n^0 + \lambda \psi_n^1 + \ldots \right) \]

Now group in like powers of \( \lambda \). The first order terms in \( \lambda \) read

\[ H^0 \psi + n^1 + H^1 \psi_n^0 = E_n^0 \psi_n^0 + E_n^1 \psi_n^0 \]

Now comes the tricky part. We want to inner product everything with \( \psi_n^0 \) to get rid of some inner products.

\[ \langle \psi_n^0 | H^0 | \psi_n^0 \rangle + \langle \psi_n^0 | H^1 | \psi_n^0 \rangle = \langle \psi_n^0 | E_n^0 \psi_n^0 \rangle + \langle \psi_n^0 | E_n^1 \psi_n^0 \rangle \]

\[ E_n^0 \langle \psi_n^0 | \psi_n^0 \rangle + \langle \psi_n^0 | H^1 | \psi_n^0 \rangle = E_n^0 \langle \psi_n^0 | \psi_n^0 \rangle + E_n^1 \langle \psi_n^0 | \psi_n^0 \rangle \]

\[ \langle \psi_n^0 | H^1 | \psi_n^0 \rangle = E_n^1 \]

To get the wavefunctions, first we start by grouping the first order terms in groups of \( \psi \)

\[ \left( H^0 - E_n^0 \right) \psi_n^1 = - \left( H^1 - E_n^1 \right) \psi_n^0 \]

We then expand \( \psi_n^1 \) as a linear combination of the eigenstates of the unperturbed Hamiltonian to deal with the \( H^0 \psi_n^1 \) term.

\[ \sum \left( E_{m}^0 - E_n^0 \right) c_{m} \psi_{m} = - \left( H^1 - E_n^1 \right) \psi_n^0 \]

Then we take the inner product with the same eigenstate \( \psi_n \).

\[ \sum \left( E_{m}^0 - E_n^0 \right) c_{m} < \psi_{m} | \psi_{n}^0 > = -< \psi_{m} | H^1 | \psi_{n}^0 > + E_n^1 < \psi_{m} | \psi_{n}^0 > \]

This allows us to simplify some of the inner products. We get rid of the \( < \psi_{n} | \psi_{n} > \) inner product by saying \( n \neq m \), which gives 0 on the RHS anyway.

\[ \left( E_{m}^0 - E_n^0 \right) c_{m} = -< \psi_{m} | H^1 | \psi_{n}^0 > \]

\[ c_{m} = < \psi_{m} | H^1 | \psi_{n}^0 > \]

\[ \psi_{n} = \sum_{m \neq n} < \psi_{m} | H^1 | \psi_{n} > \psi_{m} \]

1st order PT Energies

\[ E_n^1 = < \psi_n^0 | H_1 | \psi_n^0 > \]

1st order PT wavefunctions

\[ \psi_n^1 = \sum_{m \neq n} \psi_m \frac{< \psi_m | H_1 | \psi_n^0 >}{E_m^0 - E_n^0} \]
2nd order PT Energies

\[ E_2^* = -\sum_{\text{non-fs}} \frac{\langle H \rangle_{\text{fs}}^2}{E_2 - E_1^*} \]

Coefficient for the time dependent transition probability

\[ m_{i \rightarrow f} = -\hbar \int_0^t dt e^{i(E_f - E_i)\tau / \hbar} < f | V(t') | i > \]

Fermi’s golden rule

\[ P_{i \rightarrow f} \approx \frac{2\pi}{\hbar} |\langle f | V(t') | i \rangle|^2 \delta (E_f - E_i - \hbar \omega) \]

Fermi’s golden rule transition rate

\[ \Gamma = \frac{\hbar}{2} |\langle f | F | i \rangle|^2 \delta (\Delta E - \hbar \omega) + \frac{\hbar}{2} |\langle f | F^\dagger | i \rangle|^2 \delta (\Delta E + \hbar \omega) \]

Fermi’s golden rule density of states

\[ \Gamma_{\text{tot}} = \frac{\hbar}{2} \rho (E_i + \hbar \omega) \langle |\langle f | F | i \rangle|^2 + \frac{\hbar}{2} \rho (E_i - \hbar \omega) \langle |\langle f | F^\dagger | i \rangle|^2 \]

Fermi’s golden rule easy. Density of states

\[ R = \frac{\hbar}{2} \rho (E_i - \hbar \omega) \langle \langle f | V | i \rangle \rangle^2 + \frac{\hbar}{2} \rho (E_i + \hbar \omega) \langle \langle f | V^\dagger | i \rangle \rangle^2 \]

4.7 Variational Method

Statement about the ground energy

\[ E_0 \leq \langle \psi_\sigma | H | \psi_\sigma \rangle \]

4.8 EM in QM

Hamiltonian with magnetic and electric correction

\[ H = \langle p - qA \rangle^2 + q\phi \]

4.9 WKB

How to approximate \( \psi \) far from the turning points.

\[ p(x) = \sqrt{2m(E - V(x))} \rightarrow \psi(x) \approx \frac{1}{\sqrt{p(x)}} \exp \left( \pm \frac{\hbar}{i} \int_0^x dp(y) \right) \]

4.10 Scattering

1D scattering

Set waves on either side for scattering and reflecting, then match BC in continuity in value and slope.

First Born approximation

\[ f(\theta, \phi) = -\frac{2m}{\hbar^2} \int d^3r' e^{i(k_i - k_f) \cdot r'} V(r') \text{ then } \frac{\partial}{\partial r^i} = |f|^2 \]

First Born approximation spherical symmetry

Take \( \kappa = 2k \sin(\theta/2) \) then

\[ f(\theta, \phi) = -\frac{2m}{\hbar^2} \int d^3r' V(r') \sin(\kappa r') \]

5 Statistical Mechanics

Probability in terms of multiplicity

\[ P(n) = 1/\Omega(E) \]

Entropy definition

\[ S(E) = k \ln \Omega \]

Multiplicity of 2 systems together

\[ \Omega_{12} = \Omega_1 \Omega_2 \]

Entropy of two systems

\[ S_{12} = S_1 + S_2 \]

\[ dU = -dS + P dV + \mu dN \]

Free energy

\[ F = U - TS \]

Free energy from partition function

\[ F = -kT \ln Z \]

Gibbs Free energy

\[ G = U - TS + PV \]
Heat capacity definition \[ c = \frac{\partial E}{\partial T} \]

2nd law of thermodynamics (kind of) \[ \Delta S = \int_{T_1}^{T_2} \frac{\partial Q}{T} \, dt \]

Sterling’s approximation \[ \ln(N!) \approx N \ln N - N \]

Multiplicity in terms of Boltzmann factors \[ \Omega = \frac{1}{Z} \sum e^{-\beta E_n} \]

\[ < E > \] in terms of \( Z \) \[ < E > = U = -\frac{\partial}{\partial \beta} \ln Z \]

\[ \frac{\sigma^2}{k} \]

\[ \frac{\sigma^2}{k} = \frac{3}{2} \ln Z \]

Equipartition theorem Each particle has energy distributed evenly on average across all degrees of freedom. Total energy is \[ < U > = \frac{1}{2} kT \]

Density of states of a Fermi gas \[ g(E) = \frac{V}{4\pi^2} \frac{(\frac{\hbar^2}{2m})^{3/2}}{\sqrt{E}} \]

What is the preferred way to derive the density of states? State many expressions.

\[ \int g(\epsilon) \, d\epsilon = \int \rho d\mathbf{k} = \int \rho \frac{1}{4\pi} \, d\epsilon \]

The \( 2\pi \) comes from imposing periodic boundary conditions. It would be \( \pi \) for Dirichlet BC. In 3D, take this in polar coordinates in \( k \) space and this becomes \( \left( \frac{1}{4\pi} \right)^3 \int 4\pi k^2 dk \).

The number of particles covers up to the fermi-level in 1 octant of a sphere with 2 fold degeneracy from spins. \[ \cdot \]

For a dispersion relation, we can use \[ E = \frac{\hbar^2 k^2}{2m} \]

so \[ dE = \frac{\hbar^2}{2m} \, dk \]. Plugging this back in, we get the above expression for a density of states.

The number of particles covers up to the fermi-level in 1 octant of a sphere with 2 fold degeneracy from spins. \[ \cdot \]

Then using energies \[ E = \frac{\hbar^2 k^2}{2m} = \frac{\hbar^2}{2m} (\pi n L) \]

The wavenumber here comes from the infinite potential box condition. Then you can find that \[ \epsilon_F = \frac{\hbar^2}{2m} \left( \frac{\lambda N \pi^2}{V^2} \right)^{2/3} \]. This is also the chemical potential at 0 temperature.

Fermi-Dirac distribution \[ f_{FD} = \frac{1}{e^{\beta(\epsilon - \mu)} + 1} \]

Bose-Einstein distribution \[ f_{BE} = \frac{1}{e^{\beta(\epsilon - \mu)} - 1} \]

\( \gamma \) for degrees of freedom \[ \gamma = (f + 2)/f \]

Partition function of an ideal gas \[ Z = (V/\lambda^3)^N/N! \] where \( \lambda = \frac{h}{\sqrt{2\pi m kT}} \)

What is constant over an adiabatic curve? \[ PV^\gamma = \text{const} \]

What does adiabatic mean \[ dQ = 0 \]

What does reversible mean? \[ dS = 0 \]

What does isothermal mean \[ dU = 0 \]

Clausius Clapyron equation. State the main way to get there \[ \frac{dP}{dT} = \frac{L}{(V_{\text{gas}} - V_{\text{liquid}})} \]

Say the Gibbs energies are the same \[ G_{\text{gas}} = G_{\text{liquid}} \] then solve for \( dP/dT \)

Definition of latent heat \[ L = T(S_{\text{gas}} - S_{\text{liquid}}) \]

Carnot engine efficiency \[ \eta = 1 - Q_H/Q_C = 1 - T_C/T_H \]

Carnot fridge efficiency \[ c.o.p. = 1 - Q_H/Q_C = 1 - T_C/T_H \]
Maxwell’s relations

How does a phase transition relate to free energy

How does a phase transition relate to chemical potential

Van der waals equation condition for the critical P,V and T

\[ \frac{\partial^2 V}{\partial T^2} \bigg|_{T_c,V_c} = 0 \] 3rd equation is the equation of state

5.1 Polymers / Random Walks

What is the mean displacement of a random walker?

What is the standard deviation of a random walker?

What is the probability density function (Standard normal distribution)

Freely rotating chain variance

Work done on a polymer

5.2 Ising Model

Mean field approximation

Partition function for 1D Ising model.

Average spin definition

Magnetization in terms of distribution function

Critical Temperature condition

5.3 Brownian motion