Problem M99Q.2

(a) We are in the LS coupling regime.

The electronic wavefunction in $n = 2$ can be in a $2s$ or a $2p$ configuration, with $L = 0, 1$ respectively. The total electron spin can couple to $S = 0, 1$.

Thus we obtain possible term symbols ${}^1S_0$, ${}^3S_1$, ${}^1P_1$, ${}^3P_0$, ${}^3P_1$, ${}^3P_2$, with degeneracies $1, 3, 3, 1, 3, 5$, respectively. Here we use the spectroscopic notation ${}^{2S+1}L_J$. As expected, the total number of states is $2^2 \cdot (1 + 3) = 16$.

Since the nuclear spin is zero, there is no hyperfine coupling. Thus the quantum state is fully described by the term symbol together with the total angular momentum projection m_J.

(b) The P states will tend to have higher energy than the S states, barring relativistic corrections, since its spatial density is concentrated farther from the nucleus.

Since the singlet state is antisymmetric, the spatial wavefunction must be symmetric for the ${}^1S_0$ and ${}^1P_1$ states. Thus the two electrons will on average be closer in singlet states than in triplet states, increasing the energy of Coloumb repulsion (i.e. the *exchange term* is positive).

Thus we expect the ${}^1P_1$ state to have the highest overall energy.

(c) The $1s^2$ state of helium consists only of the ${}^1S_0$ ground state (the triplet state ${}^3S_1$ is forbidden by the Pauli exclusion principle).

The ${}^1P_1$ state may decay via an E1 transition directly to the ground state, which occurs very rapidly. The ${}^3P_0$, ${}^3P_1$, and ${}^3P_2$ states may decay via E1 transition to the $1s\,2s\,{}^3S_1$ level, with rate limited by the ω^3 spontaneous emission rate.

By parity selection, the ${}^1S_0$ and ${}^3S_1$ cannot decay via an E1 process, and are thus metastable. The ${}^1S_0$ state may decay via absorption of a virtual photon on the ${}^1S_0 \to {}^1P_1$ transition, followed by decay to the ground state. This is a two-photon (2E1) transition, with decay rate limited mainly by the ω^3 factor bottlenecking the transition to ${}^1P_1$.

The ${}^3S_1$ state must decay to the ground state, since no intermediate state exists. The primary decay mode is via an M1 transition, since the spin state transitions from the triplet to the singlet sector. This transition is highly forbidden, and so the ${}^3S_1$ state should have a very long lifetime.

The longest-lived state is likely ${}^3S_1$, and the shortest-lived state is likely ${}^1P_1$.