BRIEF ARTICLE

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Let Cylinder axis on z direction and E field points at $\hat{x}$ direction. By symmetry, potential depends on (r, θ). The solutions to Poisson equation are:

$$V(r > R) = \sum_n (A_n \cos(k_n\theta) + B_n \sin(k_n\theta)) r^{k_n} \tag{1}$$

$$V(r < R) = \sum_n (C_n \cos(k_n\theta) + D_n \sin(k_n\theta)) \frac{1}{r^{k_n}} - E_0 r \cos\theta \tag{2}$$

And their derivatives are:

$$dV/dr(r > R) = \sum_n (A_n \cos(k_n\theta) + B_n \sin(k_n\theta)) k_n r^{k_n - 1} \tag{3}$$

$$dV/dr(r < R) = \sum_n (C_n \cos(k_n\theta) + D_n \sin(k_n\theta)) \frac{-k_n}{r^{k_n+1}} - E_0 \cos\theta \tag{4}$$

$$\frac{dV}{d\theta}(r > R) = \sum_n (-A_n \sin(k_n\theta) + B_n \cos(k_n\theta)) k_n r^{k_n} \tag{5}$$

$$\frac{dV}{d\theta}(r < R) = \sum_n (-C_n \sin(k_n\theta) + D_n \cos(k_n\theta)) k_n \frac{1}{r^{k_n}} + E_0 r \cos\theta \tag{6}$$

Boundary Condition: $k_n = 1$:, matching radial part gives $A_1 = (-\epsilon_0 C_1/R^2)/\epsilon$ and $\epsilon B_1 = -D_1/R$
matching angular part gives $-A_1 R = -C_1/R + E_0 R$ and $-B_1 R = -D_1/R$
$k_n! = 1$:, radial part gives $\epsilon A_n R^{2k_n} = -C_n$ and $\epsilon B_n R^{2k_n} = -D_n$
angular part gives $A_n R^{k_n} = C_n$ and $B_n R^{2k_n} = D_n$

If $k! = 1$ and $\epsilon! = -1$, then $A_n = B_n = C_n = D_n = 0$
if $k = 1$ and $\epsilon! = -1$, then $B_1 = D_1 = 0$. And $A_1 = -E_0 + C_1/R^2 = 1/\epsilon(-E_0 - C_1/R^2) \rightarrow$ $C_1 = E_0 R^2 (\frac{\epsilon-1}{\epsilon+1})$ and $A_1 = -E_0 \frac{2}{\epsilon+1}$

Now plug everything back in:

$$V(r > R) = -E_0 r \frac{2}{\epsilon + 1} \cos\theta \tag{7}$$

$$V(r < R) = E_0 R^2 / r (\frac{\epsilon - 1}{\epsilon + 1}) \cos\theta - E_0 r \cos\theta \tag{8}$$

And E field are:

$$E(r < R) = E_0(\frac{2}{\epsilon + 1})\hat{x} \tag{9}$$

$$E(r < R) = E_0(R/r)^2(\frac{\epsilon - 1}{\epsilon + 1})(\cos\theta\hat{r} + \sin\theta\hat{\theta}) + E_0\hat{x} \tag{10}$$