5/99 EM2

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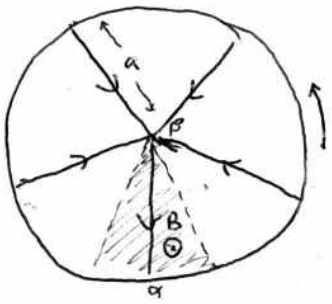


Radius a

Resistance/length λ

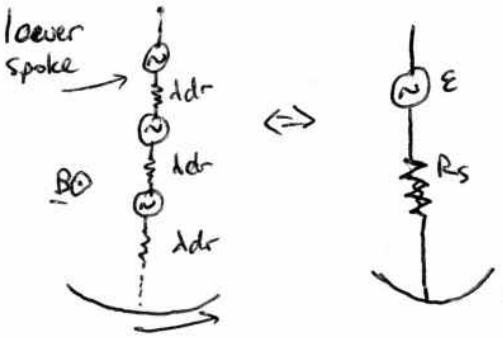
A.) Resistance of spokes: $R_s = \lambda a$

Outer segments: $R_o = \frac{2\pi a \lambda}{5}$

B.) Since the spokes are rotating at angular speed ω, electrons in the lower spoke feel $q\frac{(\omega r\hat{\theta})}{c} \times \underline{B}$ force

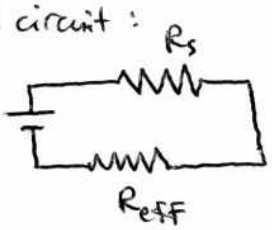
$$\mathcal{E} = \frac{1}{q}\int_{\alpha}^{\beta} q\frac{\omega r}{c}(\hat{\theta}\times\underline{B})\cdot d\underline{\ell} = \int_0^a \frac{\omega r}{c} B\, dr = \frac{\omega B a^2}{2c}$$

Equivalent circuit: The EMF acts as a distributed voltage source:

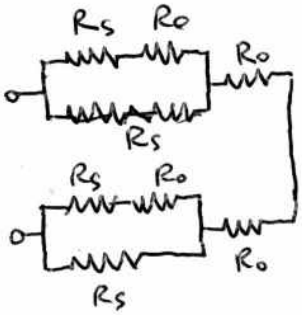


$\Leftrightarrow$

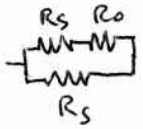
Full circuit:



R_{eff}:



let R_p:



$$\Rightarrow \frac{1}{R_{eff}} = \frac{1}{R_p+R_o} + \frac{1}{R_p+R_o} = \frac{2}{R_p+R_o}, \quad \frac{1}{R_p} = \frac{1}{R_s+R_o} + \frac{1}{R_s} = \frac{2R_s+R_o}{(R_s+R_o)R_s}$$

$$\Rightarrow \frac{1}{R_{eff}} = \frac{2}{\frac{(R_s+R_o)R_s}{2R_s+R_o} + R_o} \Rightarrow R_{eff} = \frac{1}{2}\left(\frac{R_s^2+3R_sR_o+R_o^2}{2R_s+R_o}\right), \quad R_{TOT} = R_{eff}+R_s$$

$$= \frac{5R_s^2+5R_oR_s+R_o^2}{2(2R_s+R_o)}$$

$$= \lambda a\left[\frac{5+2\pi+4\pi^2/25}{4+\frac{4\pi}{5}}\right]$$

$$= \lambda a\gamma$$

Current flowing through bottom-center spoke:

$$i = \frac{\mathcal{E}}{R_T} = \frac{\frac{\omega B a^2}{2}}{\lambda a\gamma} = \frac{\omega B a}{2\lambda\gamma}$$ (γ defined at right)

c) Moment of inertia I, initial ang. velocity ω_0

$$d\bar{\tau} = d\bar{F}\times\bar{r} = dq\,vBr = iBr\,dr \Rightarrow \tau = \frac{iBa^2}{2} = -I\dot{\omega} \Rightarrow \dot{\omega} = \frac{-\omega B^2a^4}{4IR_T} \rightarrow \frac{d\omega}{\omega} = \frac{-B^2a^4\,dt}{4R_TI}$$

$v\,dq = i\,dr$

$$\Rightarrow \omega = \omega_0 \exp\left[\frac{-B^2a^4t}{4R_TI}\right]$$