# 1 May 1998, Quantum Mechanics, Problem 2

## 1.1 (a)

The formula for the Born scattering amplitude:

$$f = -\frac{m}{2\pi}\int_0^{2\pi}\int_0^{\pi}\int_0^{\infty} e^{ik2sin(\theta/2)r'cos\theta'} V_0 e^{-(r'/a)^2} r'^2 sin\theta'\, dr'\, d\theta'\, d\phi'$$

Turn the crank and compute the integral:

$$f = -\frac{mV_0\sqrt{\pi}a^3}{2}e^{-k^2a^2sin^2(\theta/2)}$$

$$\frac{d\sigma}{d\Omega} = |f|^2 = \frac{m^2V_0^2\pi a^6}{4}e^{-2k^2a^2sin^2(\theta/2)} \tag{1}$$

## 1.2 (b)

$$\sigma = 2\pi\int_0^{\pi}\frac{d\sigma}{d\Omega}sin\theta d\theta$$

$$\sigma = m^2V_0^2\pi^2a^4\frac{e^{-k^2a^2}}{k^2}sinh(k^2a^2) \tag{2}$$

## 1.3 (c)

The usual requirement for the validity of the Born approximation is that the potential doesn't significantly affect the wavefunction, i.e., if the potential is weak or if the region of the potential is small:

$$V_0 << \frac{k^2\hbar^2}{2m} \tag{3}$$

$$a^2 << \frac{\hbar^2}{mV_0} \tag{4}$$