M98M.3

Solution to M98M.3 Fluid Dynamics

a) By applying Bernoulli principle to the top of the water and the hole, we get

\[ \frac{v_{hole}^2}{2} = \frac{v_{top}^2}{2} + gh \]  \hspace{1cm} (1)

We have flow conservation,

\[ d^2 v_{hole} = D^2 v_{top} \]  \hspace{1cm} (2)

Since \( D \gg d, v_{hole} \gg v_{top} \). The (1) becomes,

\[ \frac{v_{hole}^2}{2} = gh \]  \hspace{1cm} (3)

So

\[ v_{hole} = \sqrt{2gh} = 4.4 \text{ m/s}^2 \]  \hspace{1cm} (4)

b) There are three forces which act on the Ballon: fictitious force, gravity, buoyant force from Archimede's principle.

\[ \vec{F}_{acceleration} = -\rho_{He} V_{ballon} \, \vec{a}_x \]  \hspace{1cm} (5)

\[ \vec{P} = -\rho_{He} V_{ballon} \, \vec{g}_y \]  \hspace{1cm} (6)

\[ \vec{F}_{Archimede} = \rho_{Air} V_{ballon} \, \vec{g}_y \]  \hspace{1cm} (7)
So the angle \( \theta \) is given by,

\[
\tan(\theta) = \frac{\rho_{He} a}{(\rho_{Air} - \rho_{He})g}
\]  

(8)

c) By equaling the pressure of both sides,

\[
\frac{Mg}{\pi R^2} = \rho g H
\]  

(9)

So,

\[
H = \frac{M}{\rho \pi R^2} = 3.3cm
\]  

(10)

d) Young-Laplace equation applied to a bubble(two surfaces) gives us,

\[
\Delta p = 2\gamma \left( \frac{1}{R_x} + \frac{1}{R_y} \right)
\]  

(11)

For a sphere, \( R_x = R_y = r \). Therefore,

\[
\Delta p = \frac{4\gamma}{r} = 20 Pa
\]  

(12)

e) Navier-Stock equation for a laminar flow at stationary regime gives us:

\[
\eta \nabla v = \nabla P / \rho
\]  

(13)

Flow conservation (same section for each branch) gives us \( \sum v_i = 0 \), so that \( \sum (P_i - P_{center}) = 0 \). Therefore, \( P_{center} = (0 + P' + \alpha P' + \alpha P') = (2\alpha + 1)P' \). For having the flow outward of the fourth junction, \( P_{center} > P' \). So \( \alpha > \frac{3}{2} \).

One thought on “M98M.3”
(a) OK
(b) wrong since you forgot to add to the Archimedes force the effect of an accelerating frame.
(c) OK
(d) OK
(e) OK