M98M.3

Solution to M98M.3 Fluid Dynamics

a) By applying Bernoulli principle to the top of the water and the hole, we get

$$\frac{v_{hole}^2}{2} = \frac{v_{top}^2}{2} + gh \tag{1}$$

We have flow conservation,

$$d^2 v_{hole} = D^2 v_{top} \tag{2}$$

Since $D \gg d$, $v_{hole} \gg v_{top}$. The ((1) becomes,

$$\frac{v_{hole}^2}{2} = gh \tag{3}$$

So

$$v_{hole} = \sqrt{2gh} = 4.4m/s^2 \tag{4}$$

b) There are three forces which act on the Ballon: fictitious force, gravity, buoyant force from Archimede's principle.

$$\vec{F}_{accelaration} = -\rho_{He} V_{ballon} a \overrightarrow{e_x} \tag{5}$$

$$\vec{P} = -\rho_{He} V_{ballon} g \overrightarrow{e_y} \tag{6}$$

$$\vec{F}_{Archimede} = \rho_{Air} V_{ballon} g \overrightarrow{e_y} \tag{7}$$

So the angle θ is given by,

$$\tan(\theta) = \frac{\rho_{He} a}{(\rho_{Air} - \rho_{He})g} \tag{8}$$

c) By equaling the pressure of both sides,

$$\frac{Mg}{\pi R^2} = \rho g H \tag{9}$$

So,

$$H = \frac{M}{\rho \pi R^2} = 3.3cm \tag{10}$$

d) Young-Laplace equation applied to a bubble(two surfaces) gives us,

$$\Delta p = 2\gamma(\frac{1}{R_x} + \frac{1}{R_y}) \tag{11}$$

For a sphere, $R_x = R_y = r$. Therefore,

$$\Delta p = \frac{4\gamma}{r} = 20Pa \tag{12}$$

e) Navier-Stock equation for a laminar flow at stationary regime gives us:

$$\eta \nabla v = \nabla P / \rho \tag{13}$$

Flow conservation (same section for each branch) gives us $\sum v_i = 0$, so that $\sum (P_i - P_{center}) = 0$. Therefore, $P_{center} = (0 + P' + \alpha P' + \alpha P') = (2\alpha + 1)P'$. For having the flow outward of the fourth junction, $P_{center} > P'$. So $\alpha > \frac{3}{2}$.

One thought on “M98M.3”

December 8, 2013 at 8:44 pm

(a) OK

(b) wrong since you forgot to add to the Archimedes force the effect of an accelerating frame.

(c) OK

(d) OK

(e) OK