M98M.3

a) Applying Bernouilli's equation, taking point 1 to be the top of the container and point 2 to be at the hole, we get

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h = P_2 + \frac{1}{2}\rho v_2^2. \qquad (1)$$

$P_1 = P_2 = P_{atm}$ (the two points are at atmospheric pressure), and by the continuity equation $A_1 v_1 = A_2 v_2$ we can neglect v_1 relative to v_2, so we are left with $\rho g h = \frac{1}{2}\rho v_2^2$ or $v_2 = \sqrt{2gh}$.

b) $F = ma$ horizontally gives $\rho V a = T \sin\theta$ where V is the volume of the balloon. Vertically, $\rho V g = T\cos\theta$. Dividing the two gives $\tan\theta = \frac{a}{g}$ as the expression for the angle the balloon makes with the vertical. Note that the balloon floats in the *same* direction as the accelerating train car as the air in the train car is pushed to the back of the train, creating higher pressure, and as helium is less dense than air it will be pushed to the front.

c) $P = \frac{F}{A} = \frac{Mg}{A} = \rho g h \to h = \frac{M}{\rho A} = 0.03$ m.

d) $\Delta P = 4\frac{\gamma}{R} = 20$ Pa.

e) From the Navier-Stokes equation, consider pressure gradients along the tubes. They should cancel out in the middle (let P_c be the pressure in the center of the four tubes), giving

$$\sum \frac{(P_i - P_c)}{L} = 0 \to \sum P_i - P_c = 0 \qquad (2)$$

i.e.

$$(\alpha P' - P_c) + (\alpha P' - P_c) + (0 - P_c) + (P' - P_c) = 0 \tag{3}$$

$$\Rightarrow \frac{1}{4}(2\alpha + 1)P' = P_c \tag{4}$$

For the net flux out of the fourth tube to be outward, require that $P_c > P'$, or that $\alpha > \frac{3}{2}$.

One thought on "M98M.3"

December 2, 2013 at 10:35 pm

Ok, everything is correct.