M98M.3

a) We use Bernoulli's eqn:

$$\frac{v^2}{2} + gz + \frac{P}{\rho} = constant$$

As the diameter and height of the container is much greater than the size of the hole, we take the velocity of the water at the top of the container to be negligible.

Which leads to:

$$\frac{v^2}{2} + \frac{P_{atm}}{\rho} = gh + \frac{P_{atm}}{\rho}$$

$$v^2 = 2(9.8\,\frac{m}{s^2})(1m) \rightarrow v = 1.4\sqrt{10}\,\frac{m}{s}$$

b) We replace g with $g_{eff} =< -1, -9.8 > \frac{m}{s^2}$ which is the effective gravity for an object in the accelerating frame of the train. As $\rho_{He} < \rho_{air}$, the balloon floats in the opposite direction to g_{eff}. The balloon experiences an tension T which is balanced out by the effective gravity. As such, the balloon points forward with the acceleration of the train at an angle $\theta = tan^{-1}(\frac{1}{9.8})$

c) We use Euler's equation for hydrostatics:

$$\nabla P = \rho g \rightarrow \frac{\partial P}{\partial z} = \rho g \rightarrow P = \rho g z + P_{atm}$$

By setting the pressure on the piston ($P_{atm} + \frac{Mg}{A}$) to the pressure in the pipe $(\rho gh + P_{atm})$ equal:

$$\rho gh + P_{atm} = P_{atm} + \frac{Mg}{A} \to \rho gh = \frac{Mg}{A}$$

Resulting in:

$$h = \frac{M}{\rho A} = \frac{100kg}{\pi m^2 1g/cm^3} = \frac{10}{\pi} cm$$

d) For the differential Helmholtz free energy at constant temperature, we have

$dF = -PdV + TdA = 0$, where T is surface tension and P is the pressure difference.

This leads to $PdV = TdA$

For a sphere: $P(4\pi r^2 dr) = T(2 * 8\pi r dr) \to P = \frac{4T}{r}$. Here, I have doubled the differential area as there is an inner and outer surface to the soap bubble

Consequently, $P = \frac{4*50dynes/cm}{1cm} = 200dynes$

e) For Poiseuille flow, we have $\frac{dV}{dt} = f(R, L, \eta)|\Delta P|$. In this system, R, L, η are constant for all four tubes. Therefore, the flow in a particular pipe is proportional to the pressure difference between the pipe junction and the end of the pipe. I look for a transition point in α by setting the pressure in the junction to be P'. In order to conserve volume, I require $v_{left} + v_{right} = 2v_{left} = -v_{down}$ (using symmetry for left and right pipes). The pressure difference between the left pipe and the junction is $(1-\alpha)P'$ and between the bottom pipe and junction is P'. Setting this into our velocity relationship, we find that the only value of α that satisfy this equation is $\frac{3}{2}$. I try a few test α values. If $\alpha = 1$, symmetry between the top pipe and left pipe requires inward flow. If $\alpha = 0$, all of the pipes but the top pipe have zero pressure, so flow is inward from the top pipe. If $\alpha = \infty$, the pressure from the top pipe is negligible and must be outward. We therefore conclude that $\alpha > \frac{3}{2}$ in order for there to be outward flow from the junction in the top pipe.

One thought on “M98M.3”

October 27, 2013 at 1:23 am

Everything is correct.
Remarks:
c) assumption of smallness of change of water is not actually needed.
e) how did you get your $\alpha = 1/2$ case? I'd say that $\alpha = 3/2$ is the only possible solution. (maybe you took an absolute value of the pressure difference? then it should be just the pressure difference for an obvious physical reason)