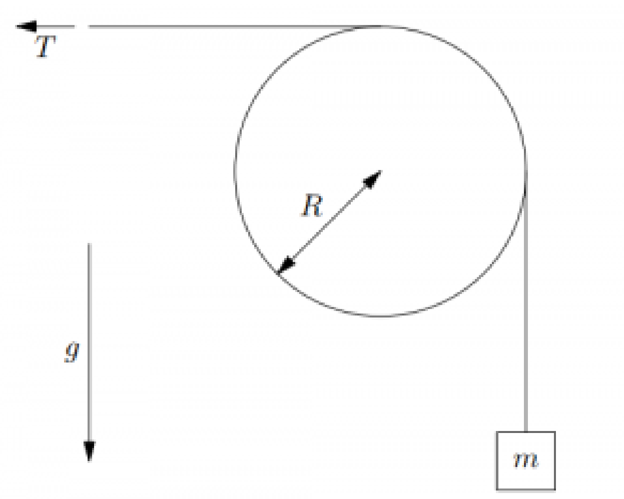
M98M.1

Problem:

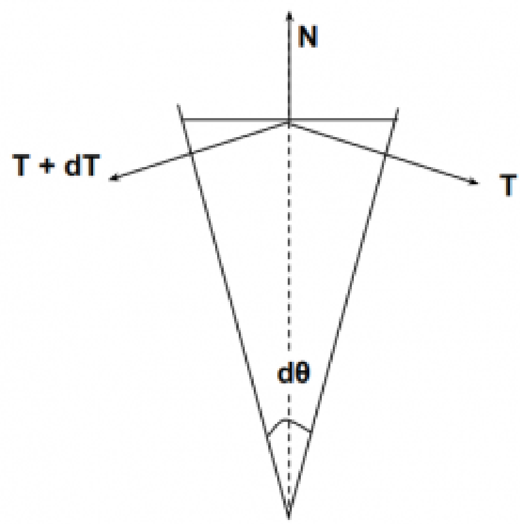
A mass m is lifted by means of a rope drawn across a cylinder as sketched in the figure. The cylinder is fixed so that it does not rotate. A steady horizontal tension T is applied, and the mass rises vertically with no acceleration. Find an expression for T in terms of the coefficient of kinetic friction, μ, between the cylinder and the rope.



Solution:

Since the mass is moving without acceleration, the tension at the end of the rope should always equal to m g. However, friction between the rope and the cylinder must be considered. Take an infinitesimal angle, $d\theta$, on the cylinder, the change in the tension force should equal to the friction force caused by the normal force.

Decompose the forces in the tangent and the normal direction, we get Figure 2.



Thus two equations can be written in each direction:

$$F_{tangent} = (T + dT)\cos\frac{\Delta\theta}{2} - T\cos\frac{\Delta\theta}{2} - \mu N_{\Delta\theta} = 0 \tag{1}$$

$$F_{normal} = (T + dT)\sin\frac{\Delta\theta}{2} + T\sin\frac{\Delta\theta}{2} - N_{\Delta\theta} = 0 \tag{2}$$

At very small $\Delta\theta$, we may take approximations:

$\sin\frac{\Delta\theta}{2} = \frac{\Delta\theta}{2} = \frac{d\theta}{2}$

$\cos\frac{\Delta\theta}{2} = 1$

Solve for dT and $d\theta$, we cancel out $N_{\Delta\theta}$ and get:

$$dT = -\mu T d\theta \tag{3}$$

Solve for T as a function of θ, we get:

$$T = T_0 \exp(-\mu\theta) \tag{4}$$

Note that when $\theta = 0, T = mg$. Thus $T_0 = mg$.

Also we know that the tension applied is equal to the tension at $\theta = \pi/2$.

Finally, we get:

$$T = mg\exp\left(-\frac{\mu\pi}{2}\right) \tag{5}$$

One thought on “M98M.1”

October 17, 2013 at 5:29 pm

Good, everything is correct.

Small comment: you write "the friction force caused by the normal force". It's not really caused by the normal force, it's just expressed through it. Example: you can have a body lying on the horizontal table -- there is a normal force but there is no friction force.