M98M.1

The tension T on the cylinder is related to θ. For each point on the rope, the force balance would be:

$$\vec{T}(\theta+\delta\theta)-\vec{T}(\theta)=\delta\vec{f}+\delta\vec{N}=\delta f\hat{\theta}+\delta N\hat{r} \tag{1}$$

where $\delta\vec{f}$ is the friction force provided by the cylinder in the azimuthal direction, and $\delta\vec{N}$ is the supporting force provided by the cylinder in the radial direction.

$$\vec{T}(\theta+\delta\theta)-\vec{T}(\theta)=\delta(T\hat{\theta})=\delta T(\theta)\hat{\theta}+T(\theta)\delta(\hat{\theta})=\delta T(\theta)\hat{\theta}+T(\theta)\delta\theta\hat{r} \tag{2}$$

$$(1)\&(2)\Rightarrow\ \delta T(\theta)=\delta f \tag{3}$$

$$T(\theta)\delta\theta=\delta N \tag{4}$$

Since the coefficient of kinetic friction μ is known, we have

$$\delta f=\mu\delta N \tag{5}$$

$$(3),(4)\&(5)\Rightarrow\ \delta T(\theta)=\mu T(\theta)\delta\theta \tag{6}$$

$$\Rightarrow\frac{\delta T(\theta)}{T(\theta)}=\mu\delta\theta \tag{7}$$

$$\Rightarrow\frac{dT}{T}=\mu d\theta \tag{8}$$

$$\Rightarrow\ln T|_{Mg}^{T_0}=\mu\theta|_0^{\frac{\pi}{2}} \tag{9}$$

$$\Rightarrow\ln(\frac{T_0}{Mg})=\frac{\pi}{2}\mu \tag{10}$$

$$\Rightarrow T_0=Mg\cdot e^{\frac{\pi}{2}\mu} \tag{11}$$

Where T_0 is the tension at left side of the cylinder. Therefore to match the denotation of the problem, we have $T = Mg \cdot e^{\frac{\pi}{2}\mu}$.

One thought on “M98M.1”

October 17, 2013 at 4:50 pm

Good, everything is correct.