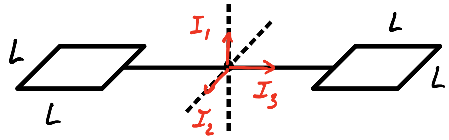
M18M.3 (Space Panels)

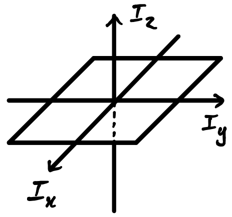
(a)



By ē symmetries of ē problem, it is clear tt ē principle axes are ē axis of ē beam, ē axis in ē plane of ē panels ⊥ to ē beam, & ē axis ⊥ to ē plane of ē panels.

Visually, it looks like we have $I_3 < I_2 < I_1$ as shown above.

First, we will find ē moments of inertia of ē planar square panel through ē centre.



⊥ axis thm: $I_z = I_x + I_y$

$$I_x = I_y = \frac{m}{L^2}\int_{-L/2}^{L/2} dx \int_{-L/2}^{L/2} dy\, y^2$$

$$= \frac{4m}{L^2}\int_0^{L/2} dx \int_0^{L/2} dy\, y^2$$

$$= \frac{4m}{L^2}\cdot\frac{L}{2}\cdot\frac{1}{3}\cdot\frac{L^3}{8}$$

$$= \frac{1}{12}mL^2 \implies I_z = \frac{1}{6}mL^2$$

Return^g to ē problem, we immediately see tt $I_3 = \frac{1}{6}mL^2$

For I_2, we apply ē // axis thm to I_x from before:

$$I_2 = 2\cdot\left(I_x + m\cdot\tfrac{1}{2}(L+d)^2\right)$$

$$= \frac{1}{6}mL^2 + mL^2 + 2mLd + md^2$$

$$= \frac{7}{6}mL^2 + md^2 + 2mLd$$

Similarly, for I_1 we apply ē // axis thm to I_z:

$$I_1 = 2\cdot\left(I_z + m\cdot\tfrac{1}{2}(L+d)^2\right)$$

$$= \frac{1}{3}mL^2 + mL^2 + 2mLd + md^2$$

$$= \frac{4}{3}mL^2 + md^2 + 2mLd$$

From this, it's clear tt $I_3 < I_2 < I_1$ as required.

(b) For c̄ pseudogravity at c̄ centre of each panel to be $\frac{g}{6}$, we must have:

$$\vec{\omega} = \frac{g}{3(L+d)}\,\hat{2}$$, where c̄ $\hat{1}, \hat{2}, \hat{3}$ axes are c̄ principal axes from before.

c̄ Euler equat^{n}s are derived as: $\left.\frac{d\vec{L}}{dt}\right|_{inertial} = \left.\frac{d\vec{L}}{dt}\right|_{rot} + \vec{\omega}\times\vec{L}$

$$\begin{cases} \tau_1 = \frac{dL_1}{dt} + \omega_2 L_3 - \omega_3 L_2 = I_1\dot{\omega}_1 + \omega_2\omega_3(I_3 - I_2) \\ \tau_2 = \frac{dL_2}{dt} + \omega_3 L_1 - \omega_1 L_3 = I_2\dot{\omega}_2 + \omega_1\omega_3(I_1 - I_3) \\ \tau_3 = \frac{dL_3}{dt} + \omega_1 L_2 - \omega_2 L_1 = I_3\dot{\omega}_3 + \omega_1\omega_2(I_2 - I_1) \end{cases} \xRightarrow{\vec{\tau}=0} \begin{cases} I_1\dot{\omega}_1 + \omega_2\omega_3(I_3 - I_2) = 0 \\ I_2\dot{\omega}_2 + \omega_1\omega_3(I_1 - I_3) = 0 \\ I_3\dot{\omega}_3 + \omega_1\omega_2(I_2 - I_1) = 0. \end{cases}$$

In this problem, we apply a small perturbat^{n} to c̄ rotat^{n} about I_2. So, we can say:

$\omega_2 \longrightarrow \omega_2 - \varepsilon$

$\omega_3 \longrightarrow \varepsilon$ (assume c̄ perturbat^{n} excites I_3 rotat^{n}).

Differentiate c̄ 3rd Euler equat^{n}: $I_3\ddot{\varepsilon} + (I_2 - I_1)(\dot{\omega}_1\omega_2 + \omega_1\dot{\omega}_2) = 0$

Drop c̄ ω_1 term: $I_3\ddot{\varepsilon} = -(I_2 - I_1)\dot{\omega}_1\omega_2$

Substitute ω_1 from c̄ 1st equat^{n}: $\ddot{\varepsilon} = -\frac{1}{I_3}(I_2 - I_1)\,\omega_2^2\,\omega_3\,\frac{1}{I_1}(I_2 - I_3)$

$$\approx \varepsilon \cdot \frac{\omega_2^2}{I_1 I_3}(I_1 - I_2)(I_2 - I_3) \equiv \Omega^2\varepsilon\ , \text{ w/ } \Omega^2 > 0.$$

Thus, this is a grow^{g} perturbat^{n}, w/ characteristic freq. $\Omega = \omega_2\sqrt{\left(1 - \frac{I_2}{I_1}\right)\left(\frac{I_2}{I_3} - 1\right)}$

$\Longrightarrow$ c̄ characteristic time is $\tau \sim \frac{1}{\Omega}$ //.