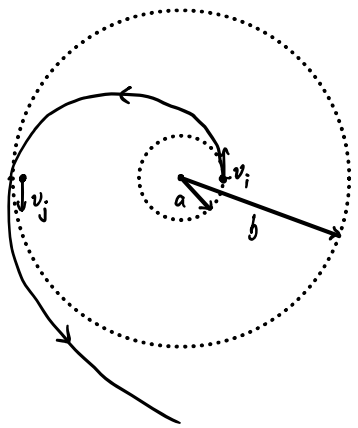
M18M.1 (Planetary Slingshot)

(a)



When ē probe is launched from Earth, it is at radius a w/ some velocity v_i, act^{d} only under ē influence of ē Sun. If travels out to a radius b, at which it has velocity v_{probe}.

First, we can solve for ē velocities of ē Earth & Jupiter in their circular orbits.

$$\frac{v_c^2}{r_c} = \frac{GM}{r_c^2} \text{ , where } M = M_\odot$$

$$\Longrightarrow v_e = \sqrt{\frac{GM}{a}} \text{ , } v_j = \sqrt{\frac{GM}{b}}$$

Focus^{s} on ē probe now, apply ē conservat^{n} of energy & angular momentum:

Energy: $\frac{1}{2}mv_i^2 - \frac{GMm}{a} = \frac{1}{2}mv_{probe}^2 - \frac{GMm}{b} \Longrightarrow v_{probe}^2 = v_i^2 + 2GM\left(\frac{1}{b} - \frac{1}{a}\right)$

Angular momentum: $mav_i = mbv_{probe} \Longrightarrow v_i = \frac{b}{a}v_{probe}$ (assum^{s} perpendicularity at launch & rendezvous).

$$\Longrightarrow v_{probe}^2\left(1 - \frac{b^2}{a^2}\right) = 2GM\left(\frac{1}{b} - \frac{1}{a}\right)$$

$$v_{probe}^2 = 2GM\left(\frac{a}{b(a+b)}\right)$$

Finally, use $v_j^2 = \frac{GM}{b}$: $v_{probe}^2 = 2v_j^2\left(\frac{a}{a+b}\right) \Longrightarrow v_{probe} = v_j\sqrt{\frac{2a}{a+b}}$

$\because b > a$, it is clear tt $v_{probe} < v_j$ ✓.

(b) $\because$ we are treat^{d} this as an elastic collis^{n}, a simple treatment us^{s} conservat^{n} of energy & momentum suffices.

We know tt in ē limit of $m_1 \gg m_2$: $v_2 = \frac{2m_1}{m_1+m_2}u_1 + \frac{m_2-m_1}{m_1+m_2}u_2$

$$\approx 2u_1 - u_2$$

$\Longrightarrow v_p' \approx v_p + 2v_j$ ✓ (ē probe receives a boost of $2v_j$).

(c) Consider ē maximal distance from ē Sun: $-\frac{GM}{r_{max}} = -\frac{GM}{b} + \frac{1}{2}v_p'^2$

$$= -\frac{GM}{b} + \frac{1}{2}(v_p + 2v_j)^2$$

$$= -v_j^2 + \frac{1}{2}v_p^2 + 2v_j^2 + v_p v_j$$

$$= \frac{1}{2}v_p^2 + v_j^2 + v_p v_j$$

$$= \frac{1}{2}(v_p + v_j)^2 + \frac{1}{2}v_j^2 > 0.$$

Thus, r_{max} can be arbitrarily large (a better condit^{n} is to say ē velocity is posit^{ve} even arbitrarily far).