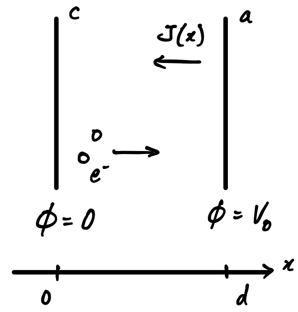
M18E.1 (Space Charge)

(a)



ē continuity equat^{n} in ē reg^{n} reads: $\frac{\partial \rho}{\partial t} + \frac{\partial J}{\partial x} = 0$

We are given ē fact that $\frac{\partial \rho}{\partial t} = 0$, so we know $\frac{\partial J}{\partial x} = 0$.

$\Longrightarrow J(x) = J_0 = \rho(x)\, u(x)$

ē Poisson equat^{n} in ē reg^{n} between ē plates gives: $\nabla^2 \phi = -\frac{\rho}{\varepsilon_0}$

$$\Longrightarrow \nabla^2 \phi = -\frac{J_0}{\varepsilon_0 u}$$

By ē conservat^{n} of energy, we also have: $\frac{1}{2} m u^2 = e\phi$ (kinetic energy gain due to accelerat^{n} through PD).

$$\Longrightarrow u(x) = \sqrt{\frac{2e}{m}}\sqrt{\phi}$$

Insert^{g} this into ē previous relat^{n}:

$$\nabla^2 \phi(x) = -\frac{J_0}{\varepsilon_0}\sqrt{\frac{m}{2e}}\,\phi^{-1/2}$$

Here, $\nabla^2 \equiv \frac{\partial^2}{\partial x^2}$, so I will assume a power law ansatz: $\phi(x) = c x^n$

$$\Longrightarrow c\,n(n-1)\,x^{n-2} = \underbrace{-\frac{J_0}{\varepsilon_0}\sqrt{\frac{m}{2e}}}_{\alpha}\,\frac{1}{\sqrt{c}}\,x^{-n/2}$$

$$\Longrightarrow c^{3/2}\, n(n-1)\, x^{n-2} = -\alpha x^{-n/2}$$

We demand ē powers are equal: $n - 2 = -\frac{n}{2} \Longrightarrow n = \frac{4}{3}$

& ē coefficients are equal: $c^{3/2}\left(\frac{4}{3}\right)\left(\frac{1}{3}\right) = -\alpha \Longrightarrow c = \left(-\frac{9\alpha}{4}\right)^{2/3}$

$\Longrightarrow \phi(x) = \left(-\frac{9\alpha}{4}\right)^{2/3} x^{4/3}$, w/ $\alpha = \frac{J_0}{\varepsilon_0}\sqrt{\frac{m}{2e}}$ ($J_0 < 0$ so it's good).

(b) ē boundary condit^{n}s are: $\phi(0) = 0$ (trivially satisfied).

$$\phi(d) = V_0 \Longrightarrow V_0 = \left(-\frac{9J_0}{4\varepsilon_0}\sqrt{\frac{m}{2e}}\right)^{2/3} d^{4/3}$$

Thus, $J_0 = -V_0^{3/2} \cdot \frac{4\varepsilon_0}{9d^2}\sqrt{\frac{2e}{m}}$ //.