Problem M18E.2

(a) We may assume point charges (i.e. $R = 0$) since the field outside the spheres is unchanged under this assumption. The dipole moment of the dumbbell is $p = Qd$, so that

$$\ddot{\mathbf{p}} = \omega^2 \mathbf{p}.$$

Then the Poynting vector at far-field is

$$\mathcal{S} = \frac{\mu_0}{16\pi^2 r^2 c} |\ddot{\mathbf{p}} \times \hat{\mathbf{r}}|^2 \hat{\mathbf{r}} = \frac{\mu_0 \omega^4}{16\pi^2 r^2 c} (Qd)^2 \sin^2\theta\, \hat{\mathbf{r}},$$

where θ is the angle between $\mathbf{p}$ and $\hat{\mathbf{r}}$. Integrating over all outgoing directions gives

$$\mathcal{P} = \boxed{\frac{\mu_0 \omega^4 Q^2 d^2}{6\pi c}}.$$

(b) The magnetic dipole moment of the magnet is $m = M\pi R^2 d$. As in part (a), the Poynting vector at far-field is

$$\mathcal{S} = \frac{\mu_0}{16\pi^2 r^2 c^3} |\ddot{\mathbf{m}} \times \hat{\mathbf{r}}|^2 \hat{\mathbf{r}} = \frac{\mu_0 \omega^4}{16\pi^2 r^2 c^3} (M\pi R^2 d)^2 \sin^2\theta\, \hat{\mathbf{r}},$$

so the total radiated power is

$$\mathcal{P} = \boxed{\frac{\mu_0 \pi \omega^4 M^2 R^4 d^2}{6 c^3}}.$$

The magnetic quadrupole moment is suppressed by an additional factor of $(v/c)^2$, and is negligible for a mechanical object.

(c) The charge on each sphere satisfies

$$\begin{aligned} V &= \int_R^\infty (\mathbf{E} \cdot \hat{\mathbf{r}})\, \mathrm{d}r \\ &= \int_R^\infty \frac{Q}{4\pi\epsilon_0 r^2}\, \mathrm{d}r \\ &= \frac{Q}{4\pi\epsilon_0 R}, \end{aligned}$$

so that

$$Q = 4\pi\epsilon_0 R V \sim 110\,\mathrm{nC}.$$

We have

$$\omega^4 = (2\pi)^4 (10\,\mathrm{kHz})^4 \sim 1.6 \times 10^{19}\,\mathrm{s}^{-1},$$

so that

$$\mathcal{P} \sim 40\,\mathrm{pW}.$$

The magnetization of the cylinder satisfies

$$M = B/\mu_0 \sim 800\,\mathrm{kA/m},$$

so that

$$\mathcal{P} \sim 24\,\mu\mathrm{W}.$$

The permanent magnet radiates about 6×10^5 times more power.