M17T.2 (Surface Adsorptⁿ)

For a free boson in 3D, we can treat it as a classical ideal gas w/ 3 translatⁿal degrees of freedom.

For an adsorbed boson in 2D, we treat it as a Bose gas w/ 2 translatⁿal degrees of freedom.

$$\varepsilon_{3D} = \frac{p^2}{2m} = \frac{p_x^2 + p_y^2 + p_z^2}{2m}$$

$$\varepsilon_{2D} = -\varepsilon_0 + \frac{p_x^2 + p_y^2}{2m}$$

$\bar{c}$ 2D Bose gas can also neglect condensatⁿ phenomena, so we cannot neglect quantum statistics.

Treat $\bar{c}$ 3D gas as a grand canonical ensemble, & $\bar{c}$ 2D gas as another. At equilibrium, we only have $\bar{c}$ requirement that $\mu_{3D} = \mu_{2D} = \mu$.

We know $\bar{c}$ canonical partitⁿ functⁿ for $\bar{c}$ classical ideal gas is: $Q(T, V) = \frac{V}{\lambda^3}$

Thus, $\bar{c}$ grand partitⁿ functⁿ is: $\mathcal{Z}(T, V, \mu) = \sum_{N=0}^{\infty} e^{\beta\mu N} \frac{1}{N!} \frac{V^N}{\lambda^{3N}}$, w/ $\lambda = \sqrt{\frac{2\pi\hbar^2}{mk_BT}}$ is $\bar{c}$ thermal wavelength.

$$= \exp\left(\frac{Vz}{\lambda^3}\right), \text{ where } z = e^{\beta\mu} \text{ is } \bar{c} \text{ fugacity.}$$

We can write $\Phi = \log \mathcal{Z} = \frac{Vz}{\lambda^3} = \frac{PV}{k_BT} \Rightarrow e^{\beta\mu} = \frac{P}{k_BT}\lambda^3 \Longrightarrow \mu = k_BT \log\left(\frac{P\lambda^3}{k_BT}\right)$

For $\bar{c}$ 2D gas, we can write $\bar{c}$ total particle number as:

$$N = \int d\varepsilon\, g(\varepsilon)\, n_{BE}(\varepsilon)$$

$$= \int d\varepsilon\, g(\varepsilon) \frac{1}{\exp(\beta(\varepsilon-\mu)) - 1}$$

$$= \int d\varepsilon \cdot g(\varepsilon) \frac{1}{\exp[\beta(\varepsilon - \varepsilon_0 - \mu)] - 1}$$

We thus need $\bar{c}$ 2D density of states. $\bar{c}$ number of states in a shell of radius p is: $N(k) = \frac{\pi k^2}{(2\pi/L)^2}$

$$\Longrightarrow N(k) = \frac{Ak^2}{4\pi} \Longrightarrow N(\varepsilon) = \frac{Am}{2\pi\hbar^2}\varepsilon$$

Thus, $\bar{c}$ density of states is simply $g(\varepsilon) = \frac{dN(\varepsilon)}{d\varepsilon} = \frac{Am}{2\pi\hbar^2}$

$$\Longrightarrow N = \frac{Am}{2\pi\hbar^2}\int d\varepsilon \frac{1}{\exp[\beta(\varepsilon - \varepsilon_0 - \mu)] - 1}$$

$\bar{c}$ surface density is simply: $\frac{N}{A} = \frac{m}{2\pi\hbar^2}\int \frac{d\varepsilon}{\exp[-\beta(\varepsilon_0+\mu)]\exp(\beta\varepsilon) - 1}$ (let $e^{-\beta(\varepsilon_0+\mu)} = \alpha$)

$$= \frac{m}{2\pi\hbar^2}\int \frac{d\varepsilon}{\alpha\exp(\beta\varepsilon) - 1} \quad \left(\text{let } \beta\varepsilon = x,\ d\varepsilon = \frac{1}{\beta}dx\right)$$

$$= \frac{m}{2\pi\hbar^2\beta}\int_0^\infty \frac{dx}{\alpha e^x - 1}$$

$$= \frac{-m}{2\pi\hbar^2\beta}\int_0^\infty \frac{dx}{1 + \eta e^x} \quad (\text{w/ } \eta = -\alpha)$$

$$\Longrightarrow \frac{N}{A} = \frac{m}{2\pi\hbar^2\beta} \log\left(\frac{1}{1-\alpha}\right)$$

Finally, we just use $\bar{\text{e}}$ express^{n} for α & μ from $\bar{\text{e}}$ 3D gas:

$$\frac{N}{A} = \frac{m}{2\pi\hbar^2\beta} \log\left(\frac{1}{1-\exp[-\beta(\varepsilon_0+\mu)]}\right)$$

$$= \frac{mk_B T}{2\pi\hbar^2} \log\left[\frac{1}{1-\exp[-\varepsilon_0/k_B T - \log(P\lambda^3/k_B T)]}\right]$$

$$= \frac{mk_B T}{2\pi\hbar^2} \log\left[\frac{1}{1-(k_B T/P\lambda^3)\exp(-\varepsilon_0/k_B T)}\right] //.$$