Prelims Solutions

Problem M15T1

Valentin Skoutnev

1

There are 2^3 possible combinations of up/down for 3 spins. This gives:

$$Z_1 = e^{-\beta(-3J-3\mu_o H)} + 3e^{-\beta(J+\mu_o H)} + 3e^{-\beta(J-\mu_o H)} + e^{-\beta(-3J+3\mu_o H)}$$

2

Notice:

$$< m >= -\frac{1}{\beta}\frac{\partial ln(Z_1)}{\partial H} = \frac{3\mu_o}{Z_1}(-e^{-\beta(-3J-3\mu_o H)} + e^{-\beta(J+\mu_o H)} - e^{-\beta(J-\mu_o H)} + e^{-\beta(-3J+3\mu_o H)})$$

Simplifying, using $Z_1 = 2(3e^{-\beta J}cosh(\beta\mu_o H) + e^{\beta 3J}cosh(3\beta\mu_o H))$:

$$< m >= -3\mu_o\frac{e^{-\beta J}sinh(\beta\mu_o H) + e^{\beta 3J}sinh(3\beta\mu_o H)}{3e^{-\beta J}cosh(\beta\mu_o H) + e^{\beta 3J}cosh(3\beta\mu_o H)}$$

3

Plug and chug.

4

We can find the entropy from the free energy:

$$F = -Tln(Z_1^N) \rightarrow S = -\frac{\partial F}{\partial T}$$

Assuming the trimers are not identical. If they are, add an $N!$ in the denominator of the $ln()$. Thus,

$$S = Nln(Z_1) + \frac{NT}{Z_1}(\frac{-1}{T^2})(\frac{\partial Z_1}{\partial \beta})$$

$$S = Nln(2(3e^{-\beta J}cosh(\beta\mu_o H) + e^{\beta 3J}cosh(3\beta\mu_o H))) + \frac{3NJ}{T}(\frac{e^{-\beta J}cosh(\beta\mu_o H) - e^{\beta 3J}cosh(3\beta\mu_o H)}{3e^{-\beta J}cosh(\beta\mu_o H) + e^{\beta 3J}cosh(3\beta\mu_o H)}) +$$
$$-\frac{3N\mu_o H}{T}(\frac{e^{-\beta J}sinh(\beta\mu_o H) + e^{\beta 3J}sinh(3\beta\mu_o H)}{3e^{-\beta J}cosh(\beta\mu_o H) + e^{\beta 3J}cosh(3\beta\mu_o H)})$$

5

J is parameter for internal coupling between spins. H is for coupling with external field.

As J goes to infinity, S goes to $Nln(2)+$ a function of(H, N, μ_o, T) which corresponds to each trimer having its internal spins aligned in one direction, up or down. N trimers gives 2^N possibles states so $S = Nln(2)$ plus the weight from the interaction with the external field.

As J goes to 0, S goes to a function of H, N, μ_o and T which corresponds to the entropy of trimers with their energy only governed by the interaction of individual spins with the external magnetic field.

As J goes to negative infinity, S goes to $Nln(6)+$ a function of(H, N, μ_o, T) which corresponds to each trimer having its internal spins anti-aligned, there are 6 way to do this. N trimers gives 6^N possibles states so $S = Nln(6)$ plus the weight from the interaction with the external field.