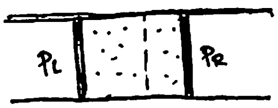
MIST. 2



Adiabatic process $\rightarrow dS = 0$ or $dQ_L + dQ_R = 0$

N is conserved

a.) As p and S is constant in this system, then we can guess that enthalpy is conserved where it is given by

$$H = U + pV$$

in the system we have

$$dH = dU_L + dU_R + p_L\, dV_L + p_R\, dV_R$$

$$= (dU_L + p_L\, dV_L) + (dU_R + p_R\, dV_R)$$

$$= dQ_L + dQ_R$$

$$= 0 \qquad \square$$

b.) $H_f = H_0$

$$U_f + p_R V_R = U_i + p_L V_L$$

$$\Delta U = p_L V_L - p_R V_R$$

c.) As $F = U - TS \Rightarrow dF = dU - TdS - dT\,S \Rightarrow S = -\left(\frac{\partial F}{\partial T}\right)_{N,V}$ we have

$$= (-p\,dV + \mu\,dN) - S\,dT$$

$$S = +Nk_B\left\{\ln\left[\left(\frac{mk_BT}{2\pi\hbar^2}\right)^{3/2}\frac{V-Nb}{V}\right]+1\right\} + Nk_BT\frac{3}{2T}$$

thus

$$U = F + TS$$

$$= -Nk_BT\left\{\ln\left[\left(\frac{mk_BT}{2\pi\hbar^2}\right)^{3/2}\frac{V-Nb}{V}\right]+1\right\} - \frac{N^2a}{V} + Nk_BT\left\{\ln\left[\left(\frac{mk_BT}{2\pi\hbar^2}\right)^{3/2}\frac{V-Nb}{V}\right]+1\right\} + \frac{3}{2}Nk_BT$$

$$= \frac{3}{2}Nk_BT - \frac{N^2a}{V}$$

d.) Consider the change of temperature in this process as a function of change in pressure

$$dH = T\,dS + dp\,V = 0$$

then as $dS = \left(\frac{\partial S}{\partial T}\right)_p dT + \left(\frac{\partial S}{\partial p}\right)_T dp$ we get :

$$= \frac{C_p}{T} dT - \left(\frac{\partial V}{\partial T}\right)_p dp$$

$$C_p dT - T\left(\frac{\partial V}{\partial T}\right)_p dp + V dp = 0$$

$$dT = \frac{dp}{C_p}\left[T\left(\frac{\partial V}{\partial T}\right)_p - V\right]$$

in this process we have $dp<0$, then for cooling where $dT<0$ we have

$$T\left(\frac{\partial V}{\partial T}\right)_p - V > 0 \quad \text{--- as } C_p > 0$$

for van der Waals we have

$$\left(p + \frac{N^2 a}{V^2}\right)(V - Nb) = Nk_B T$$

$$pV + \frac{N^2 a}{V} - pNb - \frac{N^3 ab}{V^2} = Nk_B T$$

$$V = \frac{Nk_B T}{p} - \frac{N^2 a}{pV} + Nb + \frac{N^3 ab\, p}{p^2 V^2}$$

$$V \approx \frac{Nk_B T}{p} - \frac{Na}{k_B T} + Nb + \frac{Nab\,p}{k_B^2 T^2} \quad \text{--- substituting } pV \approx Nk_B T$$

then, we have

$$\left(\frac{\partial V}{\partial T}\right)_p \approx \frac{Nk_B}{p} + \frac{Na}{k_B T^2} - 2\frac{Nab\,p}{k_B^2 T^3}$$

keeping only terms linear to a and b we get

$$T\left(\frac{\partial V}{\partial T}\right)_p - V \approx \frac{2Na}{k_B T} - Nb$$

thus the critical condition is given by

$$\frac{2a}{k_B T} - b > 0$$

$$\frac{2a}{k_B T} > b$$