M15M.1 Solution

Eric Emdee

January 10, 2017

This is classic small oscillations. First, let's write a Lagrangian.

$$
\begin{aligned}
T =& \frac{1}{2} I_\theta \dot{\theta}^2 + \frac{1}{2} I_{sphere} \dot{\phi}^2 \\
V =& -mg(R-r)\cos\theta
\end{aligned}
$$

the non-slip condition gives $\dot{\phi} r = \dot{\theta}(R-r)$. A lot of people have questioned this non-slip condition but this is definitely correct. To see why, think about the change in the contact point if the ball moved an angle θ along the cylindier, while entirely slipping. The contact point would move a distance $r\theta$. Thus when we add in rolling, the distance on the ball is $r\theta + r\phi$ while the distance on the cylinder is just $R\theta$. Equating these two and taking the derivative gives the stated condition

$$
\mathcal{L} = \frac{1}{2} m(R-r)^2 \dot{\theta}^2 + \frac{1}{5} m r^2 \dot{\theta}^2 \left(\frac{R-r}{r} \right)^2 + mg(R-r)\cos\theta
$$

From this we get the equation of motion in θ

$$
\left(m(R-r)^2 + \frac{2}{5} m r^2 \left(\frac{R-r}{r} \right)^2 \right) \ddot{\theta} = -mg(R-r)\sin\theta
$$

Small angle approximation gives us $\sin\theta \approx \theta$

$$
\begin{aligned}
\ddot{\theta} +& \frac{mg(R-r)}{m(R-r)^2 + \frac{2}{5} m r^2 \left(\frac{R-r}{r} \right)^2} \theta = 0 \\
\ddot{\theta} +& \frac{g}{(R-r) + \frac{2}{5}(R-r)} \theta = 0 \\
\Rightarrow \omega =& \sqrt{\frac{5g}{7(R-r)}}
\end{aligned}
$$