Prelims Solutions

Problem M12T2

Eric Emdee and Valentin Skoutnev

1

Maximum entropy will be attained when both reservoirs are at the same temperature T_{eq}. Because no work or heat is exchanged with the environment, $dU_{sys} = dU_1 + dU_2 = C_1 dT_1 + C_2 dT_2 = 0$. This gives for the equilibrium temperature $C_1(T_{eq} - T_{1o}) + C_2(T_{eq} - T_{2o}) = 0 \rightarrow T_{eq} = \frac{C_1 T_{1o} + C_2 T_{2o}}{C_1 + C_2}$. The total entropy increase is:

$$\Delta S = \int \frac{dQ_1}{T_1} + \int \frac{dQ_2}{T_2} = \int_{T_{1o}}^{T_{eq}} \frac{C_1 dT_1}{T_1} + \int_{T_{2o}}^{T_{eq}} \frac{C_2 dT_2}{T_2} = C_1 ln(\frac{T_{eq}}{T_{1o}}) + C_2 ln(\frac{T_{eq}}{T_{2o}})$$

2

Consider reservoir 1 expelling heat dQ_1 at temperature T_1. This heat then does some work $dW = dQ_1 - dQ_2$ and the remaining heat dQ_2 is absorbed by reservoir 2 at temperature T_2. Reversibility requires no entropy increase when transferring heat which gives $\frac{dQ_1}{T_1} = \frac{dQ_2}{T_2}$. Also, the reservoirs lose/gain energy $dU_1 = -dQ_1 = C_1 dT_1$ and $dU_2 = dQ_2 = C_2 dT_2$. Now our equation for no entropy increase becomes:

$$-\frac{C_1 dT_1}{T_1} = \frac{C_2 dT_2}{T_2} \rightarrow \int_{T_{1o}}^{T_{eq}} -\frac{C_1 dT_1}{T_1} = \int_{T_{2o}}^{T_{eq}} \frac{C_2 dT_2}{T_2} \rightarrow (\frac{T_{1o}}{T_{eq}})^{C_1} = (\frac{T_{eq}}{T_{2o}})^{C_2}$$

Hence, $T_{eq} = (T_{1o}^{C_1} T_{2o}^{C_2})^{\frac{1}{C_1 + C_2}}$.

We can get work done from $dW = -C_1 dT_1 - C_2 dT_2 \rightarrow$

$$W = -C_1(T_{eq} - T_{1o}) - C_2(T_{eq} - T_{2o}) = C_1(T_{1o} - T_{eq}) - C_2(T_{eq} - T_{2o})$$