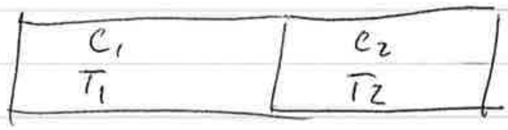
m12t2

a.



$T_1 > T_2$

conserve energy: $c_1(T_a - T_1) + c_2(T_b - T_2) = 0$

$$\Delta S_{tot} = \int_{T_1}^{T_a} \frac{C_1\,dT}{T} + \int_{T_2}^{T_b} \frac{c_2\,dT}{T}$$

$$= c_1 \log\left(\frac{T_a}{T_1}\right) + c_2 \log\left(\frac{T_b}{T_2}\right)$$

$$T_b = -\frac{c_1}{c_2}(T_a - T_1) + T_2$$

$$\Delta S_{tot}(T_a) = c_1 \log\left(\frac{T_a}{T_1}\right) + c_2 \log\left(1 - \frac{c_1(T_a - T_1)}{c_2 T_2}\right)$$

$$\frac{\partial \Delta S_{tot}}{\partial T_A} = \frac{c_1}{T_a} + c_2\left(\frac{c_2 T_2}{c_2 T_2 - c_1(T_a - T_1)}\right)\left(\frac{-c_1}{c_2 T_2}\right) = 0$$

$$\frac{1}{T_a} - \frac{c_2}{c_2 T_2 - c_1(T_a - T_1)} = 0$$

$$\frac{1}{T_a} - \frac{1}{T_b} = 0 \rightarrow T_a = T_b \text{ maximizes } S$$

(thermal equilibrium).

$$\rightarrow \Delta S_{tot} = \log\left(\frac{T_f^{(c_1+c_2)}}{T_1^{c_1} T_2^{c_2}}\right)$$

where T_f determined by $c_1(T_f - T_1) + c_2(T_f - T_2) = 0$.