m12m2

a. $\vec{F}_i = -k\vec{y}_i$

$\Sigma \vec{F} = 0$, $\quad \Sigma \vec{N} = 0$ about any point.

~~$\Sigma y_i m = 0$~~

$Mg - k\sum_{i=1}^{N} y_i = 0$

torque about CM $(x=0)$: $\quad \sum_{i=1}^{N} x_i(-ky_i) = 0 \rightarrow \sum_{i=1}^{N} x_i y_i = 0.$

~~then~~ explicitly, $\quad Mg - k\sum_{i \neq j}^{N} y_i - ky_j = 0 = \sum_{i \neq j}^{N} x_i y_i + x_j y_j$

$$-y_j(-k - x_j) = \sum_{i \neq j}(x_i + k)y_i - Mg$$

$$y_j = \frac{Mg - \sum_{i \neq j}(x_i + k)y_i}{k + x_j}$$

and $\quad F_j = -ky_j$.

b. nontwisting mode - ~~$\Sigma \vec{F} = 0$ still, +~~ $\Sigma \vec{N} = 0$ still, though $\Sigma \vec{F} \neq 0$.

guess $\quad y_i = A\cos\omega t$, then no torque condition becomes

$$\Sigma x_i y_i = 0 \rightarrow \Sigma x_i = 0.$$

twisting mode, $\quad \Sigma \vec{F} = 0$, though $\Sigma \vec{N} \neq 0$.

$\tan\theta = \frac{y_{max}}{x}$ $\quad$ so guess $\quad y_i(x_i) = A(x)\cos\omega t$ (includes sign.)

$$y_i(x_i) = x_i \tan\theta_{max}\cos\omega t + \frac{Mg}{kN}$$

no net force $\rightarrow$ $\quad Mg - k\tan\theta_{max}\cos\omega t \,\Sigma x_i + Mg = 0$

$$\rightarrow \Sigma x_i = 0 \text{ again.}$$