Prelims Solutions

Problem M10M2

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1

Because the velocity is constant, the net forces on everything must be zero. A zero velocity wouldn't allow us to say much about μ because static friction allows only bounding arguments.

Thus, the tension on the left is Mg, tension on the right is mg. Along the cylinder the tension is continuously increasing. Let θ vary from 0 to π with respect to radial line that passes through the point where the rope first touches the cylinder on the right. Analyzing the geometry of a small section the rope of width $d\theta$ on the cylinder shows that the normal force is $N = T(\theta)d\theta$. The increase in tension across this section should balance the frictional force $dT(\theta) = \mu N = \mu T(\theta)d\theta$. Solving gives $T(\pi) = T(0)e^{\mu\pi} \rightarrow Mg = mge^{\mu\pi} \rightarrow \mu = \frac{1}{\pi}ln(\frac{M}{m})$.