finish later

M10e1

a. $\epsilon = \varepsilon_0 \begin{pmatrix} 1 & \delta & \\ \delta & 1 & \\ & & 1 \end{pmatrix} \rightarrow \quad D = \epsilon E, \quad B = \mu_0 H$

In matter: Maxwell becomes

$\nabla \cdot D = \cancel{\rho_f}^{\,0} \qquad \nabla \cdot B = 0$

$\nabla \times E = -\dot{B} \qquad \nabla \times H = \cancel{J_f}^{\,0} + \dot{D}$

$\nabla \times (\nabla \times E) = -\frac{\partial}{\partial t} \nabla \times H \mu_0 = \; = -\mu_0 \frac{\partial^2 D}{\partial t^2}$

$\nabla(\nabla \cdot E) - \nabla^2 E = -\mu_0 \frac{\partial^2 D}{\partial t^2}.$

$\vec{E} = \vec{E}_0 e^{i(kz - \omega t)} \qquad \vec{k} = k\hat{z}.$

$\vec{D} = \varepsilon_0 \begin{pmatrix} 1 & \delta & \\ \delta & 1 & \\ & & 1 \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} = \varepsilon_0 \begin{pmatrix} E_x + \delta E_y \\ \delta E_x + E_y \\ E_z \end{pmatrix}$

$\rightarrow \nabla(i\vec{k} \cdot \vec{E}) - (ik)^2 \vec{E} = -\mu_0 (-i\omega)^2 \epsilon \vec{E}$

$-\vec{k}(\vec{k} \cdot \vec{E}) + k^2 \vec{E} = \omega^2 \mu_0 \varepsilon_0 \vec{E}$

$-k^2 E_z \hat{z} + k^2 \vec{E}_0 = \omega^2 \mu_0 \epsilon \vec{E}$

$\rightarrow \begin{pmatrix} k^2 E_x \\ k^2 E_y \\ 0 \end{pmatrix} = \frac{\omega^2}{c^2} \begin{pmatrix} E_x + \delta E_y \\ \delta E_x + E_y \\ E_z \end{pmatrix}$

so $E_z = 0$ and

$\left(k^2 - \frac{\omega^2}{c^2}\right) E_x = \frac{\omega^2}{c^2} \delta E_y$

$\left(k^2 - \frac{\omega^2}{c^2}\right) E_y = \frac{\omega^2}{c^2} \delta E_x$

$\rightarrow \left(k^2 - \frac{\omega^2}{c^2}\right)^2 \frac{c^2 \delta}{\omega^2} E_y = \frac{\omega^2}{c^2} \delta E_y$

$\left(k^2 - \frac{\omega^2}{c^2}\right)^2 = \left(\frac{\omega^2 \delta}{c^2}\right)^2$

so $k^2 - \frac{\omega^2}{c^2} = \pm \frac{\omega^2}{c^2} \delta$

$k^2 = \frac{\omega^2}{c^2}(1 \pm \delta)$

$K_+ \rightarrow \frac{\omega^2}{c^2} \delta E_x = \frac{\omega^2}{c^2} \delta E_y$ so $\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$. $\quad K_- \rightarrow \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$

$|+\rangle$ and $|-\rangle$ propagate at different k_+, k_-, though this washes out when applying BC at $z=0$.

$$\tilde{E} = \frac{E}{\sqrt{1+a^2}}$$

$$|+\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\\0\end{pmatrix} \qquad |-\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\\0\end{pmatrix}$$

$$|I\rangle = \begin{pmatrix}1\\0\\0\end{pmatrix} = \frac{1}{\sqrt{2}}\left(|+\rangle + |-\rangle\right)$$

possible phase between $|+\rangle$ and $|-\rangle$

$z<0$

$$\vec{E} = E_i\left(|+\rangle + |-\rangle\right)e^{i(kz-\omega t)} + E_R\left(|+\rangle + a|-\rangle\right)e^{i(-kz-\omega t)}$$

$$\vec{B} = \frac{E_i}{c}\left(\hat{z}\times(|+\rangle + |-\rangle)\right)e^{i(kz-\omega t)} + \frac{\tilde{E}_R}{c}\left(-\hat{z}\times(|+\rangle + a|-\rangle)\right)e^{i(-kz-\omega t)}$$

$z>0$

$$\vec{E} = E_T\left(|+\rangle + b|-\rangle\right)e^{i(k_2 z-\omega t)}$$

$$\vec{B} = \frac{\tilde{E}_T}{c}\left(\hat{z}\times(|+\rangle + b|-\rangle)\right)e^{i(k_2 z-\omega t)}$$

BC: $\epsilon_0 E_<^{\parallel} = \epsilon E_>^{\parallel}$ at $z=0$.

$$\epsilon_0\left[(E_i+E_R)|+\rangle + (E_i + aE_R)|-\rangle\right] = \epsilon_0\left(E_T(1+\Delta)|+\rangle + E_T(1-\Delta)b|-\rangle\right)$$

① $\rightarrow$ $(E_i+E_R)|+\rangle + (E_i+aE_R)|-\rangle = E_T(1+\Delta)|+\rangle + bE_T(1-\Delta)|-\rangle$

BC: $B_<^{\parallel} = B_>^{\parallel}$

$$E_i\left(\hat{z}\times(|+\rangle + |-\rangle)\right) + \frac{E_R}{\sqrt{1+a^2}}\left(-\hat{z}\times(|+\rangle + a|-\rangle)\right) = \frac{E_T}{\sqrt{1+b^2}}\left(\hat{z}\times(|+\rangle + b|-\rangle)\right)$$

$\cancel{E_i(-|-\rangle) + \frac{E_R}{\sqrt{1+a^2}}}$

② $\rightarrow$ $E_i\left(|+\rangle - |-\rangle\right) + \frac{E_R}{\sqrt{1+a^2}}\left(|-\rangle - a|+\rangle\right) = \frac{E_T}{\sqrt{1+b^2}}\left(b|+\rangle - |-\rangle\right)$

$|+\rangle$ and $|-\rangle$ are ~~lin. indep.~~ orthogonal, so we can group terms.

① $(E_i + E_R) = E_T(1+\Delta)$, $(E_i + aE_R) = bE_T(1-\Delta)$

② $E_i - \frac{aE_R}{\sqrt{1+a^2}} = \frac{bE_T}{\sqrt{1+b^2}}$, $-E_i + \frac{\cancel{a}E_R}{\sqrt{1+a^2}} = -\frac{E_T}{\sqrt{1+b^2}}$

presumably, solving this system (4 vars, 4 eqn) for a will yield the solution, though I don't feel like doing it.