

mo 9, 2

$$H = \frac{\lambda}{2} \sigma_x$$

eigenstates:  $\begin{matrix} +\lambda/2 & +\lambda/2 \\ \begin{pmatrix} 1 \\ 1 \end{pmatrix} & \begin{pmatrix} 1 \\ -1 \end{pmatrix} \\ -\lambda/2 & -\lambda/2 \end{matrix}$

note  $\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{1}{2}(x_+ + x_-)$

$$x(t) = \frac{1}{2} \left( \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-i\lambda t} + \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{i\lambda t} \right)$$

$$P_I = \left| \left\langle \begin{pmatrix} 0 \\ 1 \end{pmatrix} \middle| x(t) \right\rangle \right|^2 = \left| \frac{1}{2} (e^{-i\lambda t} - e^{i\lambda t}) \right|^2$$
$$= \sin^2 \lambda t$$

$$P_{II} = \sin^2 \lambda \left( \frac{t}{2} \right) \left| \left\langle \begin{pmatrix} 0 \\ 1 \end{pmatrix} \middle| \frac{1}{2} \left[ \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-i\lambda \frac{t}{2}} - \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{i\lambda \frac{t}{2}} \right] \right\rangle \right|^2$$
$$+ (1 - \sin^2 \lambda \frac{t}{2}) \left| \left\langle \begin{pmatrix} 0 \\ 1 \end{pmatrix} \middle| \frac{1}{2} \left[ \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-i\lambda \frac{t}{2}} + \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{i\lambda \frac{t}{2}} \right] \right\rangle \right|^2$$
$$= \sin^2 \frac{\lambda t}{2} \left| \frac{e^{-i\lambda \frac{t}{2}} + e^{i\lambda \frac{t}{2}}}{2} \right|^2 + \cos^2 \frac{\lambda t}{2} \left| \frac{e^{-i\lambda \frac{t}{2}} - e^{i\lambda \frac{t}{2}}}{2} \right|^2$$
$$= \sin^2 \frac{\lambda t}{2} \cos^2 \frac{\lambda t}{2} + \cos^2 \frac{\lambda t}{2} \sin^2 \frac{\lambda t}{2}$$
$$= \frac{\sin^2 \lambda t}{2}$$