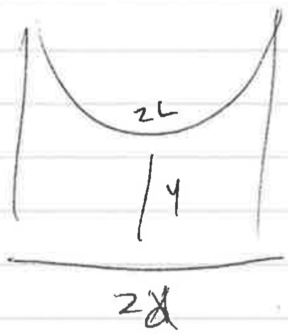
MO9 M2

a.



$$\int y\sqrt{1+y'^2}\,dx \quad \text{s.t.} \quad 2\int_0^d \sqrt{1+y'^2}\,dx = 2L$$

x-independent, so "Hamiltonian" is constant.

$$y\sqrt{1+y'^2} - \frac{y'^2 y}{\sqrt{1+y'^2}} = c$$

$$\frac{y}{\sqrt{1+y'^2}} = c$$

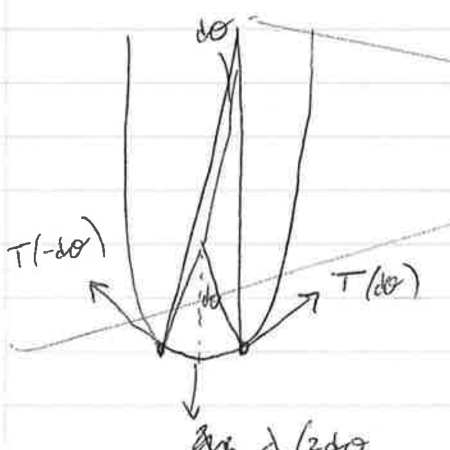
$$y = A\cosh\left(\frac{x}{A}\right) \qquad y'^2 = \sinh^2 x \rightarrow \text{checks.}$$

to determine A,

$$\int_0^d \sinh\left(\frac{x}{A}\right)dx = L$$

$$A\cosh\left(\frac{d}{A}\right) = L$$

b. Tension, let $d \ll L$.



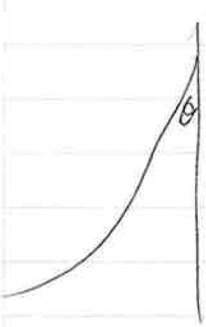
w length of bot: $2\int_0^{\delta'}\sqrt{1+y'^2}\,dx$

$= A\cosh\frac{\delta}{A}$

where $d\theta \approx \frac{\delta}{y(d)-y(\delta)} = \frac{\delta}{A\cosh\left(\frac{d}{A}\right) - A\cosh\left(\frac{\delta}{A}\right)}$

$= \frac{\delta}{L - A\left(1+\frac{\delta}{A}\right)}$

horizontal tension is constant through catenary. at each end $T_y = \frac{Mg}{2}$



$$\theta = \frac{dy}{dx} = \sinh\left(\frac{d}{A}\right)$$

this gives tension at top. which then gives horz. tension, which is only tension at bottom.

$$T_y = T\sin\theta \approx T\theta = T\sinh\left(\frac{d}{a}\right) = \frac{Mg}{2}$$

$$T = \frac{Mg}{2\sinh\left(\frac{d}{a}\right)}$$

$$\text{so } T_x = T\cos\theta = T\left(1+\frac{\theta^2}{2}\right)$$

$$T_x = \left(\frac{Mg}{2\sinh\left(\frac{d}{a}\right)}\right)\left(1 + \frac{1}{2}\sinh^2\left(\frac{d}{a}\right)\right)$$

same throughout.