close ideas ~~but not quite right.~~

mo9 m1

$$\frac{\partial \rho}{\partial t} + \nabla\cdot\rho\vec{v} = 0. \qquad \rho\frac{d\vec{v}}{dt} = -\nabla P + \cancel{\mu^2}\nabla^2\vec{v} + \vec{f}$$

incompressible $\rightarrow \nabla\cdot\vec{v} = 0.$

$\vec{v}$ is radial. Pressure outside bubble fixed.

~~In 3D: $\nabla\cdot v = \frac{1}{H}\partial_R\left(\frac{H v_R}{h_R}\right) \rightarrow \partial_r v_r = 0$~~

~~$\vec{v} = v_\theta(r)\hat{r}$.~~

~~As bubble expands, does $W = PdV = P4\pi R(t)\,dR$ on the fluid.~~

~~and also $\vec{v}(t) = \frac{dr}{dt}\hat{r}$ since this is what it is at boundary.~~

In 3D, $\nabla\cdot V = \frac{1}{r^2}\partial_r\left(r^2 v_r\right) = 0 \rightarrow 2rv_r + r^2\frac{\partial v_r}{\partial r} = 0$

so $rv' + 2v = 0$ is the velocity field diffeq.

try $v = Ar^{-2}$ $\quad -\frac{2}{r^2} + \frac{2}{r^2} = 0\,\checkmark$ where $v(r=R) = \frac{dR}{dt}$, so $v(r) = \frac{dR}{dt}\frac{R^2}{r^2}\hat{r}$

Kinetic energy of fluid = total expansion work done by bubble.

~~$$\int_R \rho v^2 d^3\vec{r} = \frac{4\pi R^3}{3}P$$~~

~~$$4\pi\rho\int_R^\infty r^2 dr\left(\frac{dR}{dt}\frac{R^2}{r^2}\right)^2 = \frac{4\pi R^3}{3}P$$~~

~~$$\rho\left(\frac{dR}{dt}\right)^2 = \frac{P}{3}$$~~ + — expansion, − — collapsing

~~$$\frac{dR}{dt} = -\frac{P}{3\rho}$$~~ is constant?

$$\frac{4}{3}\pi\left(R_{max}^3 - R^3\right)P = \frac{1}{2}\rho\int_R^\infty v^2 = 2\pi\rho\left(\frac{dR}{dt}\right)^2 R^4\int_R^\infty \frac{r^2}{r^4}dr = 2\pi\rho\left(\frac{dR}{dt}\right)^2 R^3$$

$$\longrightarrow 2\pi\rho\left(\frac{dR}{dt}\right)^2 R^3 + \frac{4\pi P}{3}R^3 = \frac{4\pi}{3}R_{max}^3 P = \text{constant} = E$$

$$\frac{dR}{dt} = \pm\sqrt{\frac{E - \frac{4\pi}{3}PR^3}{2\pi\rho R^3}} = \pm\sqrt{\frac{E}{2\pi\rho R^3} - \frac{2P}{3\rho}} = \pm\sqrt{\frac{2}{3\rho}\left(\frac{R_{max}}{R}\right)^3 P - \frac{2P}{3\rho}}$$

$$\frac{dR}{dt} = \pm\sqrt{\frac{2P}{3\rho}\left(\left(\frac{R_{max}}{R}\right)^3 - 1\right)} \quad \text{when } R = \frac{R_{max}}{2},\ \frac{dR}{dt} = \sqrt{\frac{14P}{3\rho}}.$$