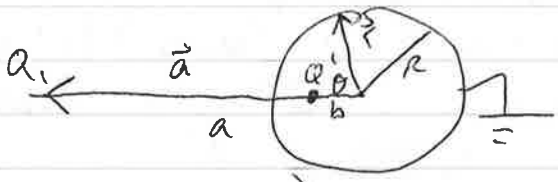
M09e3

a.



let $Q' = kQ$

$$\frac{1}{4\pi\varepsilon_0}\left(\frac{Q_1}{|\vec{a}-\vec{r}|} + \frac{Q'}{|\vec{b}-\vec{r}|}\right) = 0$$

$$\frac{1}{\sqrt{a^2+R^2-2aR\cos\theta}} - \frac{k}{\sqrt{b^2+R^2-2bR\cos\theta}} = 0$$

$$\rightarrow \quad a^2+R^2-2aR\cos\theta = b^2/k^2 + R^2/k^2 - \frac{2bR}{k^2}\cos\theta$$

thus $a = \frac{b}{k^2}$, and $a^2 = \frac{R^2}{k^2}$ and $R^2 = \frac{b^2}{k^2}$ is a solution

so

$$a = b\left(\frac{R^2}{b^2}\right) = \frac{R^2}{b} \rightarrow b = \frac{R^2}{a}$$

and $a = \frac{R^2}{ak^2} \rightarrow k = -\frac{R}{a}$ } position and magnitude of image charge.

b. conductor - equipotential, though nonzero now with more charge/no longer grounded. to change the potential, place 2nd image charge $\Delta Q = Q - Q'$ at origin. Note that physically, $\rho = 0$ within shell, so $\nabla^2 V = 0$ inside. since $V =$ constant on shell, $V =$ constant ~~throughout~~ inside as well, by uniqueness theorem.

$$V_{outside} = \left(\frac{Q_1}{|\vec{a}-\vec{r}|} + \frac{Q'}{|\vec{b}-\vec{r}|} + \frac{\Delta Q}{r}\right)\frac{1}{4\pi\varepsilon_0}$$

but $V_{inside} = C$

boundary condition: $\left.\frac{\partial V_{out}}{\partial r}\right|_R - \left.\frac{\partial V_{in}}{\partial r}\right|_R = \frac{-\sigma}{\varepsilon_0} \rightarrow \left.\frac{\partial V_{out}}{\partial r}\right|_R = \frac{-\sigma(\theta)}{\varepsilon_0}$

$$4\pi\varepsilon_0\frac{\partial V_{out}}{\partial r} = \frac{-Q_1}{(\vec{a}-\vec{r})^2}\left(2r-2a\cos\theta\right) - \frac{Q'}{(\vec{b}-\vec{r})^2}\left(2r-2b\cos\theta\right) - \frac{\Delta Q}{r^2} = \frac{-\sigma(\theta)}{\varepsilon_0}$$

$$(4\pi\varepsilon_0)\frac{\sigma(\theta)}{\varepsilon_0} = \frac{Q_1(2R-2a\cos\theta)}{a^2+R^2-2aR\cos\theta} - \frac{\frac{a}{R}Q_1\left(2R-2\frac{R^2}{a}\cos\theta\right)}{R^4/a^2+R^2-2R^2\cos\theta} + \frac{\left(Q-\frac{a}{R}Q_1\right)}{R^2}$$

(can be further simplified but this is correct)