

$m_0 c^2$

$$\frac{d}{dt} \left( \frac{1}{2} I \omega^2 + U_{em} \right) = 0$$

$$I \omega \dot{\omega} = -P_{rad} = - \frac{\mu_0 m_0^2 \omega^4}{12\pi c^3}$$

$$\dot{\omega} = - \frac{\mu_0 m_0^2}{12\pi c^3 I} \omega^3 = -\alpha \omega^3$$

$$\text{where } m_0 = M \frac{4\pi a^3}{3}$$

$$\text{solve: } \frac{d\omega}{dt} = -\alpha \omega^3 \quad \frac{d\omega}{\omega^3} = -\alpha dt \quad -\frac{1}{2\omega^2} = -\alpha t + C$$

$$\omega = \frac{1}{\sqrt{2(\alpha t + C)}} \quad \text{such that } \omega(0) = \omega_0$$

$$\omega_0 = \frac{1}{\sqrt{2C}} \rightarrow C = \frac{1}{2\omega_0^2}$$

$$\omega(t) = \frac{1}{\sqrt{2\alpha t + \frac{1}{\omega_0^2}}} \quad \alpha = \frac{\mu_0 M^2}{12\pi c^3 I} \left( \frac{4\pi a^3}{3} \right)^2 = \frac{4\mu_0 M^2 a^6 \pi}{27c^3 I}$$