Prelims Solutions

Problem M08T3

Valentin Skoutnev

1

The work function is defined to be the energy above the fermi energy that is needed for an electron to leave the surface. The flux of electrons that can get out from a wall parallel to the yz plane is given by the perpendicular velocity times the density of electrons at that velocity (given by the Fermi Dirac distribution):

$$J = \frac{1}{V}(\frac{m^2L^2}{\hbar^2\pi^2})^{\frac{3}{2}}\frac{1}{4}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{\frac{1}{2}mv_x^2=\mu+\phi}^{\infty} dv_y dv_z dv_x \frac{v_x}{e^{\beta(E-\mu)}+1}$$

where $E = \frac{1}{2}mv_x^2 + \frac{1}{2}mv_y^2 + \frac{1}{2}mv_z^2$, and normalization is calculated from $E_i = \frac{1}{2}mv_i^2 = \frac{\hbar^2\pi^2n_i^2}{2mL^2}$ (n_i needs to be positive). At room temperature the fermi dirac distribution is almost a step function and we can approximate $\frac{1}{e^{\beta(E-\mu)}+1} \approx e^{\beta\mu}e^{-\beta E}$. Carrying out the integrals I get:

$$J = \frac{2^{\frac{1}{2}}m}{\hbar^3\pi^2}(k_bT)^2 e^{\frac{-\phi}{k_bT}}$$