Prelims Potaoe Solutions

Problem M08T2

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1

Ideal gas assumes infinitesimally small potatoes and no interactions. The b parameter corrects for the finite effective size of potaoes. b is the effective volume of a potatoe, so the $V - Nb$ term is the effective unoccupied volume of the container (sack of potatoes). The a term corrects for weak interactions between potatoes. For attractive interactions, we would expect the pressure on the walls to be slightly lower since potatoes don't make it to the wall with as much speed so the we expect the pressure to be reduced by $af(N/V)$ with $a > 0$ and f some function of density.

2

Define $v = V/N$ so

$$P = \frac{kT}{(v-b)} - a\frac{1}{v^2}$$

To reach the critical point (with N fixed) we can vary P,T, and/or V. Now for a stable gas the compressibility $\frac{\partial P}{\partial V}$ must be negative. Otherwise decreasing the volume would decrease the pressure, leading to an implosion. At a critical point, the PV curve develops an inflection, which if followed through would lead to $\frac{\partial P}{\partial V}$ going from negative positive (which is avoided by part of the gas undergoing a phase transition into a liquid (mashed potatoes)). We also have $\frac{\partial^2 P}{\partial V^2} > 0$. So at the critical point

$$\frac{\partial P}{\partial v} = 0 = \frac{-kT}{(v-b)^2} + 2a\frac{1}{v^3} \to \frac{kT}{(v-b)^2} = 2a\frac{1}{v^3}$$

$$\frac{\partial^2 P}{\partial v^2} = 0 = \frac{2kT}{(v-b)^3} - 6a\frac{1}{v^4} \to \frac{kT}{(v-b)^3} = \frac{3a}{v^4}$$

Dividing the two gives $v - b = \frac{2}{3}v \to v_c = 3b$. $n_c = \frac{1}{v_c} = \frac{1}{3b}$.
Now $kT_c = \frac{2a(v_c-b)^2}{v_c^3} = \frac{8a}{27b}$.
Lastly, $P_c = \frac{(8a/27b)}{(3b)-b} - a\frac{1}{(3b)^2} = \frac{a}{27b^2}$.