Prelims Solutions

Problem M07T3

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1

The partition function is that of N independent 1/2 spins:

$$Z = (e^{-\beta(-M-B)} + e^{-\beta(M+B)})^N = (2cosh(\beta(M+B)))^N$$

$$F = -k_B T ln(Z) = -Nk_B T ln(2cosh(\beta(M+B)))$$

$$S = -\frac{\partial F}{\partial T} = Nk_B ln(2cosh(\beta(M+B))) - \frac{Nk_B(M+B)}{T} tanh(\beta(M+B))$$

$$< \Sigma_i^N \sigma_i >= N < \sigma_i >= -k_B T \frac{\partial ln(Z)}{\partial(-M)} = N tanh(\beta(M+B)) \rightarrow$$

$$\rightarrow < \sigma_i >= tanh(\beta(M+B)) = tanh(\beta(6J < \sigma_i > +B))$$

the self-consistency condition for$< \sigma_i >$.

2

$$F(< \sigma_i >! = 0) = -Nk_B T ln(2) - Nk_B T ln(cosh(\beta(6J < \sigma_i > +B))) < F(< \sigma_i >= 0) = -Nk_B T ln(2)$$

since $cosh(\beta(6J < \sigma_i > +B)) > 1$, yup.

3

The self-consistency condition $< \sigma_i >= tanh(\beta(M+B)) = tanh(\beta(6J < \sigma_i >))$ $(B = 0)$ always has $< \sigma_i >= 0$ as a solution, but the two non-zero solutions of $< \sigma_i >$ will exist only if the slope of $y = tanh(\beta(6J < \sigma_i >))$ at $< \sigma_i >= 0$ is greater than the slope of $y =< \sigma_i >$, i.e. 1. (Imagine plotting $y =< \sigma_i >$ and $y = tanh(\beta(6J < \sigma_i >))$ on a y vs $< \sigma_i >$ plot, there will either be 1 or 3 intersections/solutions). Hence, $\beta 6J > 1 \rightarrow T_c = \frac{6J}{k_B}$.