The partition function is that of $N$ independent $1/2$ spins:

$$Z = (e^{-\beta(-M-B)} + e^{-\beta(M+B)})^N = (2\cosh(\beta(M + B)))^N$$

$$F = -k_B T \ln(Z) = -N k_B T \ln(2\cosh(\beta(M + B)))$$

$$S = -\frac{\partial F}{\partial T} = N k_B \ln(2\cosh(\beta(M + B))) - \frac{N k_B (M + B)}{T} \tanh(\beta(M + B))$$

$$\langle \sum_i^N \sigma_i \rangle = N \langle \sigma_i \rangle = -k_B T \frac{\partial \ln(Z)}{\partial (-M)} = N \tanh(\beta(M + B)) \to$$

$$\langle \sigma_i \rangle = \tanh(\beta(M + B)) = \tanh(\beta(6J < \sigma_i > + B))$$

the self-consistency condition for $\langle \sigma_i \rangle$.

2

$$F(\langle \sigma_i \rangle = 0) = -N k_B T \ln(2) - N k_B T \ln(\cosh(\beta(6J < \sigma_i > + B)))) < F(\langle \sigma_i \rangle = 0) = -N k_B T \ln(2)$$

since $\cosh(\beta(6J < \sigma_i > + B))) > 1$, yup.

3

The self-consistency condition $\langle \sigma_i \rangle = \tanh(\beta(M + B)) = \tanh(\beta(6J < \sigma_i >))$ $(B = 0)$ always has $\langle \sigma_i \rangle = 0$ as a solution, but the two non-zero solutions of $\langle \sigma_i \rangle$ will exist only if the slope of $y = \tanh(\beta(6J < \sigma_i >))$ at $\langle \sigma_i \rangle = 0$ is greater than the slope of $y = \langle \sigma_i \rangle$, i.e. 1. (Imagine plotting $y = \langle \sigma_i \rangle$ and $y = \tanh(\beta(6J < \sigma_i >))$ on a $y$ vs $\langle \sigma_i \rangle$ plot, there will either be 1 or 3 intersections/solutions ). Hence, $6J > 1 \to T_c = \frac{6J}{k_B}$. 