1 May 2007, Thermodynamics, Problem 2

1.1 (a)

$$dU = TdS + Fdx$$
$$TdS = dU - \sigma l dx \tag{1}$$

1.2 (b)

$$dU = \left(\frac{\partial U}{\partial T}\right)_x dT + \left(\frac{\partial U}{\partial x}\right)_T dx = TdS + \sigma l dx$$
$$T\left(\frac{\partial S}{\partial x}\right)_T + \sigma l = \left(\frac{\partial U}{\partial x}\right)_T$$
$$dF = -SdT + \sigma l dx$$
$$l\left(\frac{\partial \sigma}{\partial T}\right)_x = -\left(\frac{\partial S}{\partial x}\right)_T \quad Maxwell\, relation$$
$$\left(\frac{\partial \sigma}{\partial T}\right)_x = -\sigma_0 a$$
$$\left(\frac{\partial S}{\partial x}\right)_T = l\sigma_0 a$$
$$\left(\frac{\partial U}{\partial x}\right)_T = Tl\sigma_0 a + \sigma l = l\sigma_0(1 + aT_0)$$
$$dU = l\sigma_0(1 + aT_0)dx \tag{2}$$
$$dQ = TdS = dU - \sigma l dx = \sigma_0 a T l dx \tag{3}$$

The positive sign means that we have to heat the system in order to keep the constant temperature.

1.3 (c)

$$dU = \left(\frac{\partial U}{\partial T}\right)_x dT + \left(\frac{\partial U}{\partial x}\right)_T dx = TdS + \sigma l dx$$
$$dT = \frac{dU - \left(\frac{\partial U}{\partial x}\right)_T dx}{\left(\frac{\partial U}{\partial T}\right)_x} = \frac{dU - l\sigma_0(1 + T_0 a)dx}{C_x}$$
$$dQ = TdS = dU - \sigma l dx = 0$$
$$dU = \sigma l dx$$
$$dT = \frac{-l\sigma_0 a T dx}{C_x} \tag{4}$$

Notice the positive sign.

1.4 (d)

The two isotherms are constant tension lines as well, so they appear horizontal on a $\sigma - x$ diagram. The adiabats are curves, and their shape can be calculated roughly:

$$\begin{aligned} \sigma &\sim -T \\ -d\sigma \sim -dT &= \frac{aT\sigma_0}{C_x}dx \sim \frac{a\sigma\sigma_0}{C_x}dx \\ \frac{d\sigma}{\sigma} &\sim -dx \\ ln\sigma &\sim -x \\ \sigma &\sim e^{-x} \end{aligned} \tag{5}$$

Therefore, the adiabats are decaying exponentials.