May 2007 #2 (QM)

scattering; Born Approximation

a. $V(\vec{r}) = \begin{cases} -V_0 & |\vec{r}| < R \\ 0 & |\vec{r}| > R \end{cases}$ incident momentum $p_0 \equiv \hbar k$

$p_0 >> \frac{\hbar}{R}$ $\quad k >> \frac{1}{R}, \quad R >> \lambda$

$$\frac{d\sigma}{d\Omega} = |f|^2; \quad f = \frac{-m}{2\pi\hbar^2}\int d\vec{r}\,' \, e^{i(\vec{k}_f - \vec{k}_i)\cdot\vec{r}\,'} V(\vec{r}\,')$$

$\vec{k}_f = k\hat{r} \qquad \vec{k}_i = k\hat{z}$

spherically symmetric potential: let the z' axis be along $\vec{q} = \vec{k}_f - \vec{k}_i$

$\Rightarrow q^2 = k^2(\hat{r}-\hat{z})^2 = k^2(1 - 2\cos\theta + 1) = 4k^2\sin^2\frac{\theta}{2}$

$q = 2k\sin\frac{\theta}{2}$

$e^{i\vec{q}\cdot\vec{r}\,'} = e^{iqr'\cos\theta'}$

$$f = \frac{-m}{\hbar^2}\int_0^\infty dr' r'^2 V(r') \underbrace{\int_{-1}^{1} d(\cos\theta)\, e^{iqr'\cos\theta'}}_{\frac{1}{iqr'}\left(e^{iqr'} - e^{-iqr'}\right) = \frac{2\sin qr'}{qr'}}$$

$$f = \frac{-2m}{\hbar^2 q}\int dr' \, r'\sin(qr')V(r') = \frac{2mV_0}{\hbar^2 q}\int_0^R dr' \, r'\sin(qr')$$

$+r' \quad \sin qr'$

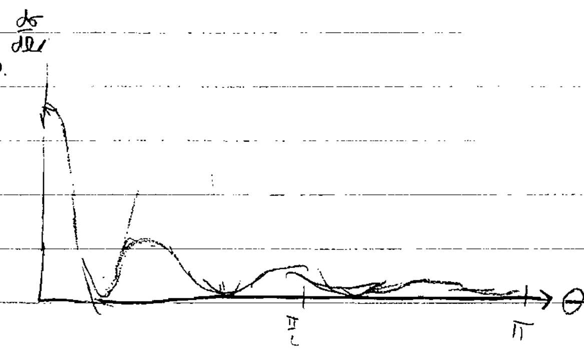
$-1 \quad -\frac{1}{q}\cos qr'$

$0 \quad -\frac{1}{q^2}\sin qr'$

$$-\frac{r}{q}\cos qr' + \frac{1}{q^2}\sin qr' \Big|_0^R = -\frac{R}{q}\cos qR + \frac{1}{q^2}\sin qR$$

$$f = \frac{-2mV_0}{\hbar^2 q^3}\left(qR\cos qR - \sin qR\right)$$

$$\frac{d\sigma}{d\Omega} = \frac{4m^2V_0^2}{\hbar^4}\frac{(qR\cos qR - \sin qR)^2}{q^6} \qquad q = 2k\sin\frac{\theta}{2}$$

b. 

Several zeros, maxima

c.