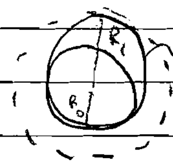
May 2007 #1 (CM)

a. Radius R_0, period T_0 satellite

Time to transfer to new circular orbit at R_1?

Hohmann Transfer:



transfer ellipse: $2a_t = R_0 + R_1$

$a_t = \frac{R_0+R_1}{2}$ semimajor axis

$T^2 = \frac{4\pi^2 \mu}{k} a^3$ T= period of ellipse a = semimajor axis

$\mu = \frac{m_s M_E}{m_s + M_E} \approx m_s$ $k = G m_s M_E$ $\frac{\mu}{k} \approx \frac{1}{GM_E}$

T is the entire period; transfer time is half the period

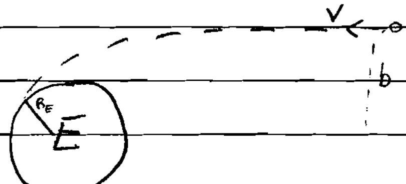
$t = \frac{\pi}{\sqrt{GM_E}} a_t^{3/2} = \frac{\pi}{\sqrt{GM_E}} \left(\frac{R_0+R_1}{2}\right)^{3/2}$

In the circular orbit, $a = R_0$ $T_0^2 = \frac{4\pi\mu R_0^3}{k}$ $\frac{4\pi\mu}{k} = \frac{T_0^2}{R_0^3}$

$T^2 = \frac{T_0^2}{R_0^3} a^3$

$$\boxed{t = \frac{1}{2} T_0 \left(\frac{R_0+R_1}{2R_0}\right)^{3/2}}$$

b.



ℓ = constant

initially, $\ell = mbv$

Total energy conserved: $\frac{1}{2}mv^2 - \frac{GmM}{r} = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2) - \frac{GmM}{r}$

$E = \frac{1}{2}m\dot{r}^2 + \frac{\ell^2}{2mr^2} - \frac{GmM}{r}$

at $r \to \infty$, $\dot{r} = v$, $E = \frac{1}{2}mv^2$

$\frac{1}{2}mv^2 = \frac{1}{2}m\dot{r}^2 + \frac{m^2b^2v^2}{2mr^2} - \frac{GmM}{r}$

For the asteroid at large radius which just barely touches the earth (largest b for which collision occurs), $\dot{r}=0$ at $r=R$

$$\frac{1}{2}mv^2 = \frac{mb^2v^2}{2R^2} - \frac{GmM}{R}$$

solving for b: $\frac{1}{2}v^2 + \frac{GM}{R} = \frac{1}{2}\left(\frac{b^2v^2}{R^2}\right)$ $\qquad \frac{b^2v^2}{R^2} = v^2 + \frac{2GM}{R}$

$$b^2 = R^2 + \frac{2GMR}{v^2}$$

$$b = R\left(1 + \frac{2GM}{v^2R}\right)^{1/2}$$

$$\# = \sigma \cdot n = \pi b^2 \cdot n = n\pi\left(R^2 + \frac{2GMR}{v^2}\right)$$