1 May 2006, Thermodynamics, Problem 3

1.1 (a)

We understand the equilibrium condition to mean that the chemical potentials of the two gases are equal. So we start by computing the chemical potential of the classical gas:

$$Z_1 = \int e^{-(p_x^2+p_y^2+p_z^2)/2mT}\,\frac{dp_x\,dp_y\,dp_z\,dx\,dy\,dz}{(2\pi)^3} = \left(\frac{mT}{2\pi}\right)^{3/2} V$$

$$Z = \frac{Z_1^N}{N!} = \left(\frac{mT}{2\pi}\right)^{3N/2} \frac{V^N}{N!}$$

$$F = -TlnZ = T\left[-\frac{3N}{2}ln\left(\frac{mT}{2\pi}\right) - NlnV + NlnN - N\right]$$

$$\mu = \left(\frac{\partial F}{\partial N}\right)_{T,V} = Tln\left[\frac{P}{T}\left(\frac{2\pi}{mT}\right)^{3/2}\right]$$

Now let's compute the total number of particles in the quantum gas. This is given by:

$$N = \int_0^\infty g(\omega)\bar{n}_{BE}(\omega)\,d\omega = \int_0^\infty g(\omega)\frac{1}{e^{(\omega-\epsilon_0-\mu)/T}-1}\,d\omega = \int_0^\infty \frac{mA}{2\pi}\frac{1}{e^{-(\epsilon_0+\mu)/T}e^{\omega/T}-1}\,d\omega$$

$$N = \int_0^\infty \frac{mAT}{2\pi}\frac{-1}{-e^{-(\epsilon_0+\mu)/T}e^x+1}\,dx = -\frac{mAT}{2\pi}\left[ln\left|\frac{e^x}{1-e^{-(\epsilon_0+\mu)/T}e^x}\right|\right]_0^\infty$$

$$N = -\frac{mAT}{2\pi}\left[\frac{\mu}{T} + \frac{\epsilon_0}{T} + ln\left(1-e^{-(\epsilon_0+\mu)/T}\right)\right]$$

Now invoke the equality of the chemical potentials. We plug in, for μ, the potential found for the classical gas:

$$\frac{N}{A} = \frac{mT}{2\pi}\left[ln\left(\frac{T}{P}\left(\frac{mT}{2\pi}\right)^{3/2}\right) - \frac{\epsilon_0}{T} - ln\left(1-e^{-\epsilon_0/T}\frac{T}{P}\left(\frac{mT}{2\pi}\right)^{3/2}\right)\right]$$

The negative signs could potentially cause trouble, but we'll hope that everything works out just right.