Ok, I have a better answer now.

You've got to be a little more careful when you're playing with continuous spectra. In analogy with what you know from the case of the position operator, you should have that the probability to measure an observable A to have values in the interval $[\alpha, \alpha + d\alpha]$ if the state vector is $|\psi >$ is given by

$$P(a \in [\alpha, \alpha + d\alpha]) = | < \alpha|\psi > |^2 d\alpha$$

provided that the eigenstates $|\alpha >$ are normalized to a delta function:

$$< \alpha'|\alpha >= \delta(\alpha' - \alpha)$$

You can check that for momentum eigenstates, you can achieve this by choosing

$$\psi_p(x) = (1/\sqrt{2\pi\hbar})e^{ipx/\hbar}$$

so that's what you should use when you're calculating the matrix elements of V_{int}.

The next problem is that your amplitude contains a delta function already – and squaring a delta function is not a good idea (the square of a distribution is not a well-defined object). The short answer is just assume that the square of the delta function is the same delta function – this should work in analogy with the position case, where you'd want the probability density for a state$|x_0 >$ to look like a delta-function, $P(x) = \delta(x - x_0)$. This is because you want probabilities to be normalized to 1.

If you want to make things better defined, you can assume the particle started in a superposition of momentum eigenstates at$t = -\infty$, and then you'll have to integrate over p_i when you calculate α_1, and thus get rid of the delta-function before squaring.

At the end of all your calculations, you'll of course want to integrate over all p_f to make sure you calculate the probability to end up in any state where particle 2 is in the first excited state of the SHO, so you should get rid of all delta functions.

Tibi