M06Q.2

(a)

The output state of photon i is given by:

$$\left|\psi_{f}
ight
angle_{i}=\delta_{1i}|\!\leftrightarrow\!
angle_{i}rac{1}{\sqrt{2}}\left(i|c
angle_{i}+|d
angle_{i}
ight)+\delta_{2i}|\!\!\downarrow
angle_{i}rac{1}{\sqrt{2}}\left(|c
angle_{i}+i|d
angle_{i}
ight)$$

(b)

In order to the write the total output state of the 2 photons, we take into account the symmetrizaton requirement of 2 identical bosuns with the appropriate normalization factor, which gives us:

$$egin{aligned} |\Psi
angle &= rac{1}{\sqrt{2}} \left(ig|\psi_fig
angle_1 ig|\psi_fig
angle_2 + ig|\psi_fig
angle_2 ig|\psi_fig
angle_1
ight) \ &= rac{1}{2\sqrt{2}} \Big(iigl|\leftrightarrowigr
angle_1 igl|\updownarrowigr
angle_2 igl|c
angle_2 - igl|\leftrightarrowigr
angle_1 igl|c
angle_1 igr|\downarrowigr
angle_2 igl|d
angle_2 + igl|\leftrightarrowigr
angle_1 igl|d
angle_1 igr|\updownarrowigr
angle_2 igl|d
angle_2 + igl|d
angle_1 igr|\updownarrowigr
angle_2 igl|d
angle_2 + igl|d
angle_2 igr|d
angle_2 igr|d
angle_2 + igl|d
angle_2 igr|d
a$$

(c)

The probability of measuring the photons leaving on opposite of the mirror is:

$$P(ext{photons leave on opposite side}) = 4 \left(rac{1}{2\sqrt{2}}
ight)^2 = rac{1}{2}$$

(d)

After the measurement, the wave function collapses to:

$$egin{aligned} |\Psi
angle &=rac{1}{2}\Big(-|\leftrightarrow
angle_1|\updownarrow
angle_2|d
angle_2+|\leftrightarrow
angle_1|d
angle_1|\updownarrow
angle_2|c
angle_2+(1\leftrightarrow2)\Big) \ &=rac{1}{2}\Big(|\leftrightarrow
angle_2|\updownarrow
angle_1-|\leftrightarrow
angle_1|\updownarrow
angle_2\Big)\Big(|c
angle_1|d
angle_2-|c
angle_2|d
angle_1\Big) \end{aligned}$$

Therefore, the polarization state of the photons is given by:

$$|\chi\rangle = |\leftrightarrow\rangle_2 |\updownarrow\rangle_1 - |\leftrightarrow\rangle_1 |\updownarrow\rangle_2$$

(e)

The density matrix of photon 1 is given by:

$$ho_1 = \sum_n ra{n}_2 (|\chi
angle \langle \chi|) |n
angle_2$$

where $|n\rangle_2$ are the polarization basis vectors of photon 2.

Performing the sum and using the orthogonality of the basis vectors, we obtain:

$$ho_1 = \left(\left| \updownarrow \right\rangle_1 \left\langle \updownarrow \right|_1 + \left| \leftrightarrow
ight
angle_1 \left\langle \leftrightarrow \right|_1
ight)$$

One thought on "M06Q.2"



December 11, 2013 at 7:28 pm

Good.

I have no comments.