M06Q.2

(a)

The output state of photon i is given by:

$$\left|\psi_f\right\rangle_i = \delta_{1i}|\leftrightarrow\rangle_i \frac{1}{\sqrt{2}}\left(i|c\rangle_i + |d\rangle_i\right) + \delta_{2i}|\updownarrow\rangle_i \frac{1}{\sqrt{2}}\left(|c\rangle_i + i|d\rangle_i\right)$$

(b)

In order to the write the total output state of the 2 photons, we take into account the symmetrizaton requirement of 2 identical bosuns with the appropriate normalization factor, which gives us:

$$\begin{aligned}|\Psi\rangle &= \frac{1}{\sqrt{2}}\left(\left|\psi_f\right\rangle_1\left|\psi_f\right\rangle_2 + \left|\psi_f\right\rangle_2\left|\psi_f\right\rangle_1\right) \\ &= \frac{1}{2\sqrt{2}}\Big(i|\leftrightarrow\rangle_1|c\rangle_1|\updownarrow\rangle_2|c\rangle_2 - |\leftrightarrow\rangle_1|c\rangle_1|\updownarrow\rangle_2|d\rangle_2 + |\leftrightarrow\rangle_1|d\rangle_1|\updownarrow\rangle_2|c\rangle_2 \\ &\qquad + i|\leftrightarrow\rangle_1|d\rangle_1|\updownarrow\rangle_2|d\rangle_2 + (1\leftrightarrow 2)\Big)\end{aligned}$$

(c)

The probability of measuring the photons leaving on opposite of the mirror is:

$$P(\text{photons leave on opposite side}) = 4\left(\frac{1}{2\sqrt{2}}\right)^2 = \frac{1}{2}$$

(d)

After the measurement, the wave function collapses to:

$$
\begin{aligned}
|\Psi\rangle &= \frac{1}{2}\left(-|\leftrightarrow\rangle_1|c\rangle_1|\updownarrow\rangle_2|d\rangle_2 + |\leftrightarrow\rangle_1|d\rangle_1|\updownarrow\rangle_2|c\rangle_2 + (1\leftrightarrow 2)\right) \\
&= \frac{1}{2}\left(|\leftrightarrow\rangle_2|\updownarrow\rangle_1 - |\leftrightarrow\rangle_1|\updownarrow\rangle_2\right)\left(|c\rangle_1|d\rangle_2 - |c\rangle_2|d\rangle_1\right)
\end{aligned}
$$

Therefore, the polarization state of the photons is given by:

$$|\chi\rangle = |\leftrightarrow\rangle_2|\updownarrow\rangle_1 - |\leftrightarrow\rangle_1|\updownarrow\rangle_2$$

(e)

The density matrix of photon 1 is given by:

$$\rho_1 = \sum_n \langle n|_2(|\chi\rangle\langle\chi|)|n\rangle_2$$

where $|n\rangle_2$ are the polarization basis vectors of photon 2.

Performing the sum and using the orthogonality of the basis vectors, we obtain:

$$\rho_1 = \left(|\updownarrow\rangle_1\langle\updownarrow|_1 + |\leftrightarrow\rangle_1\langle\leftrightarrow|_1\right)$$

One thought on “M06Q.2”

December 11, 2013 at 7:28 pm

Good.
I have no comments.