Gubser Group

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M06Q.1

a)

The potential in question is

$$V(x) = -\frac{A}{x} \tag{1}$$

While this potential reminds us of a Coulomb-like potential, since this problem is in one-dimension, we expect the wave-function to vanish at the infinite barrier at $x = 0$. From this, we guess the solution

$$\psi = Bxe^{-x/a}. \tag{2}$$

Substituting this into the Schroedinger equation

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} - \frac{A}{x}\psi = E\psi \tag{3}$$

results in

$$-\frac{\hbar^2 B}{2ma}\left(-2e^{-x/a} + \frac{x}{a}e^{-x/a}\right) - ABe^{-x/a} = BExe^{-x/a} \tag{4}$$

From this, we see that we want to choose a such that

$$\frac{\hbar^2 B}{ma}e^{-x/a} - ABe^{-x/a} = 0 \tag{5}$$

or rather

$$\frac{\hbar^2}{ma} - A = 0 \tag{6}$$

which means

$$a = \frac{\hbar^2}{mA} \tag{7}$$

From here,

$$E = -\frac{\hbar^2}{2ma^2} = -\frac{mA^2}{2\hbar^2} \tag{8}$$

b)

First, let's normalize the wavefunction.

$$\langle\psi|\psi\rangle = \int_0^\infty B^2 x^2 e^{-2x/a} dx = B^2 \frac{\partial^2}{\partial\alpha^2} \int_0^\infty e^{-x\alpha} dx = \frac{\partial^2}{\partial\alpha^2} \frac{1}{\alpha} B^2 = \frac{B^2 a^3}{4} = 1 \tag{9}$$

so $B = 2/\sqrt{a^3}$. The expectation value of x is given by

$$\langle\psi|x|\psi\rangle = \int_0^\infty B^2 x^3 e^{-2x/a} dx = -B^2 \frac{\partial^3}{\partial\alpha^3} \int_0^\infty e^{-x\alpha} dx = \frac{3}{8} B^2 a^4 = \frac{3}{2} a \tag{10}$$
$$= \frac{3\hbar^2}{2mA}$$

One thought on “M06Q.1”

December 11, 2013 at 9:47 pm

Good.

You just lost the factor of 2 in the exponent in (9) which affected the numerical factors in subsequent formulas.