The potential in question is

\[ V(x) = -\frac{A}{x} \]  

(1)

While this potential reminds us of a Coulomb-like potential, since this problem is in one-dimension, we expect the wave-function to vanish at the infinite barrier at \( x = 0 \). From this, we guess the solution

\[ \psi = Bx e^{-x/a}. \]  

(2)

Substituting this into the Schroedinger equation

\[ -\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} - \frac{A}{x} \psi = E\psi \]  

(3)

results in

\[ -\frac{\hbar^2 B}{2ma} \left( -2e^{-x/a} + \frac{x}{a} e^{-x/a} \right) - AB e^{-x/a} = BExe^{-x/a} \]  

(4)

From this, we see that we want to choose \( a \) such that

\[ \frac{\hbar^2}{ma} e^{-x/a} - AB e^{-x/a} = 0 \]  

(5)

or rather
\[
\frac{\hbar^2}{ma} - A = 0 \tag{6}
\]

which means

\[
a = \frac{\hbar^2}{mA} \tag{7}
\]

From here,

\[
E = -\frac{\hbar^2}{2ma^2} = -\frac{mA^2}{2\hbar^2} \tag{8}
\]

b)

First, let's normalize the wavefunction.

\[
\langle \psi | \psi \rangle = \int_0^\infty B^2 x^2 e^{-2x/a} dx = B^2 \frac{\partial^2}{\partial \alpha^2} \int_0^\infty e^{-x\alpha} dx = \frac{\partial^2}{\partial \alpha^2} \frac{1}{\alpha} B^2 = \frac{B^2 a^3}{4} = 1 \tag{9}
\]

so \( B = 2/\sqrt{a^3} \). The expectation value of \( x \) is given by

\[
\langle \psi | x | \psi \rangle = \int_0^\infty B^2 x^3 e^{-2x/a} dx = -B^2 \frac{\partial^3}{\partial \alpha^3} \int_0^\infty e^{-x\alpha} dx = \frac{3}{8} B^2 a^4 = \frac{3}{2} a \tag{10}
\]

\[
= \frac{3\hbar^2}{2ma}
\]

---

One thought on “M06Q.1”

December 11, 2013 at 9:47 pm

Good.
You just lost the factor of 2 in the exponent in (9) which affected the numerical factors in subsequent formulas.