1 May 2006, Electromagnetism, Problem 3

1.1 (a)

For the prelims, it is necessary to remember the Larmor formula for the total power radiated by an accelerating charge:

$$P = \frac{2}{3}\frac{q^2 a^2}{4\pi\epsilon_0 c^3}$$

It turns out that the angular distribution of the power can be obtained by reverse engineering. Notice that:

$$P = \int S_{rad}(\theta, \phi) r^2 sin\theta \, d\theta \, d\phi$$

If we assume azimuthal symmetry, then the phi integral can be computed:

$$P = 2\pi \int S_{rad}(\theta) r^2 sin\theta \, d\theta$$

Now we take a guess: if the θ-dependence were $sin^2\theta$, what would we get?

$$P = 2\pi \int_0^\pi S_0 sin^3\theta r^2 \, d\theta = 2\pi S_0 r^2 \frac{4}{3}$$

$$S_0 = P\frac{3}{8\pi r^2} = \frac{1}{4\pi r^2}\frac{q^2 a^2}{4\pi\epsilon_0 c^3}$$

$$S = \frac{1}{4\pi r^2}\frac{q^2 a^2}{4\pi\epsilon_0 c^3} sin^2\theta \tag{1}$$

This is not at all a rigorous proof (perhaps not even a proof), but it's a useful trick for the exam.

1.2 (b)

I'm not sure I understand why, but Silviu Pufu says it should be linearly polarized. It may be useful to know that electric field in order to decide this. There's no clear way to reverse-engineer this problem, so we just post the field and hope that we will remember it in the exam:

$$\boldsymbol{E}_{rad} = \frac{q}{4\pi\epsilon_0 c^2}\frac{1}{r}[\hat{r}(\hat{r}\cdot\boldsymbol{a}) - \boldsymbol{a}]$$

In all of these calculations, note that $\boldsymbol{r}$ points from the particle's position at the retarded time to the test point.

1.3 (c)

The total power radiated is given by the Larmor formula:

$$P = \frac{2}{3}\frac{a^2 e^2}{4\pi\epsilon_0 c^3}$$

$$\ddot{r} = -\frac{Ze^2}{4\pi\epsilon_0 m r^2}$$

$$P = \frac{2}{3}\frac{e^2}{4\pi\epsilon_0 c^3}\left(\frac{Ze^2}{4\pi\epsilon_0 m r_e^2}\right)^2 \tag{2}$$

where r_e is the distance between the nucleus and the electron.