M05Q.3

We can start by rewriting the Hamiltonian:

$$H/\hbar = \frac{Q_x + Q_y}{2} I^2 + \frac{Q_x - Q_y}{4}\left(I_+^2 + I_-^2\right) + \frac{2Q_z - Q_x - Q_y}{2} I_z^2$$

So we can get the Hamiltonian matrix under spinor basis with $|I, I_z\rangle$:

$$|1,1\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}; |1,0\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}; |1,-1\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

In these basises the Hamiltonian is:

$$\begin{pmatrix} \frac{2Q_z + Q_x + Q_y}{2} & 0 & \frac{Q_x - Q_y}{2} \\ 0 & Q_x + Q_y & 0 \\ \frac{Q_x - Q_y}{2} & 0 & \frac{2Q_z + Q_x + Q_y}{2} \end{pmatrix}$$

Then we can get the energy eigenvalues:

1)$E_1/\hbar = Q_x + Q_y$,with eigenfunction: $|1,0\rangle$

2)$E_2/\hbar = Q_y + Q_z$,with eigenfunction: $\frac{1}{\sqrt{2}}\left(|1,1\rangle - |1,-1\rangle\right)$

3)$E_3/\hbar = Q_z + Q_x$,with eigenfunction: $\frac{1}{\sqrt{2}}\left(|1,1\rangle + |1,-1\rangle\right)$

Then the sub-problem a) is solved;

For the second sub-problem b), we have$Q_x = Q_y = -Q, Q_z = 2Q$,the Hamiltonian can be written as $H/\hbar = H_0/\hbar + H_1/\hbar$, in which:

$$H_0/\hbar = \begin{pmatrix} Q & 0 & 0 \\ 0 & -2Q & 0 \\ 0 & 0 & Q \end{pmatrix}$$

So the last two energy levels degenerated. On the other hand the perturbation term can be written as:

$$H_1/\hbar = \frac{\omega_x - i\omega_y}{2} I_+ + \frac{\omega_x + i\omega_y}{2} I_- + \omega_z I_z$$

Then in matrix form:

$$H_1/\hbar = \begin{pmatrix} \omega_z & \frac{\omega_x + i\omega_y}{\sqrt{2}} & 0 \\ \frac{\omega_x - i\omega_y}{\sqrt{2}} & 0 & \frac{\omega_x + i\omega_y}{\sqrt{2}} \\ 0 & \frac{\omega_x - i\omega_y}{\sqrt{2}} & -\omega_z \end{pmatrix}$$

For the 2)$|1,1\rangle$ and 3)$|1,-1\rangle$ states, the 1st order perturbation of energy has ω_z with ω nonvanishing,

However, for the 1)$|1,0\rangle$, has the 1st order perturbation of energy to be 0, we nead to compute for the 2rd order perturbation, scince the other two energy levels are degenerated, and $\langle 1,1|H_1|1,-1\rangle = 0$, which we have:

$$E_n^{(2)} = -\sum_{m\neq n} \frac{|\langle m|H_1|n\rangle|^2}{E_m^{(0)} - E_n^{(0)}}$$

So the first nonvanishing perturbation of energy for $|1,0\rangle$ is:

$$E_{|1,0\rangle}^{(2)}/\hbar = -\frac{\omega_x^2 + \omega_y^2}{3Q}$$

One thought on “M05Q.3”

December 13, 2013 at 1:17 am

Everything looks correct.
Notice that you got lucky that the perturbation was diagonal on the degenerate subspace spanned by $|1,1\rangle$ and $|1,-1\rangle$. Otherwise you would have to first diagonalize the perturbation on this subspace to find the energy correction.