We can start by rewriting the Hamiltonian:

\[
\frac{H}{\hbar} = \frac{Q_x + Q_y}{2} I^2 + \frac{Q_x - Q_y}{4} \left( I_+^2 + I_-^2 \right) + \frac{2Q_z - Q_x - Q_y}{2} I_z^2
\]

So we can get the Hamiltonian matrix under spinor basis with \(|I, I_z\rangle\):

\[
|1, 1\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}; |1, 0\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}; |1, -1\rangle = \begin{pmatrix} 0 \\ 0 \end{pmatrix}
\]

In these bases the Hamiltonian is:

\[
\begin{pmatrix}
\frac{2Q_z + Q_x + Q_y}{2} & 0 & \frac{Q_x - Q_y}{2} \\
0 & Q_x + Q_y & 0 \\
\frac{Q_x - Q_y}{2} & 0 & \frac{2Q_z + Q_x + Q_y}{2}
\end{pmatrix}
\]

Then we can get the energy eigenvalues:

1) \(E_1/\hbar = Q_x + Q_y\), with eigenfunction: \(|1, 0\rangle\)

2) \(E_2/\hbar = Q_y + Q_z\), with eigenfunction: \(\frac{1}{\sqrt{2}} (|1, 1\rangle - |1, -1\rangle)\)

3) \(E_3/\hbar = Q_z + Q_x\), with eigenfunction: \(\frac{1}{\sqrt{2}} (|1, 1\rangle + |1, -1\rangle)\)
Then the sub-problem a) is solved;

For the second sub-problem b), we have \( Q_x = Q_y = -Q, Q_z = 2Q \), the Hamiltonian can be written as \( H/\hbar = H_0/\hbar + H_1/\hbar \) in which:

\[
H_0/\hbar = \begin{pmatrix}
Q & 0 & 0 \\
0 & -2Q & 0 \\
0 & 0 & Q
\end{pmatrix}
\]

So the last two energy levels degenerated. On the other hand the perturbation term can be written as:

\[
H_1/\hbar = \frac{\omega_x - i\omega_y}{2} I_+ + \frac{\omega_x + i\omega_y}{2} I_- + \omega_z I_z
\]

Then in matrix form:

\[
H_1/\hbar = \begin{pmatrix}
\omega_z & \frac{\omega_x + i\omega_y}{\sqrt{2}} & 0 \\
\frac{\omega_x - i\omega_y}{\sqrt{2}} & 0 & \frac{\omega_x + i\omega_y}{\sqrt{2}} \\
0 & \frac{\omega_x - i\omega_y}{\sqrt{2}} & -\omega_z
\end{pmatrix}
\]

For the 2) \( |1, 1\rangle \) and 3) \( |1, -1\rangle \) states, the 1st order perturbation of energy has \( \omega_z \) with \( \omega \) nonvanishing.

However, for the 1) \( |1, 0\rangle \), has the 1st order perturbation of energy to be 0, we need to compute for the 2nd order perturbation, since the other two energy levels are degenerated, and \( \langle 1, 1 | H_1 | 1, -1 \rangle = 0 \), which we have:

\[
E_n^{(2)} = -\sum_{m \neq n} \frac{|\langle m | H_1 | n \rangle|^2}{E_m^{(0)} - E_n^{(0)}}
\]

So the first nonvanishing perturbation of energy for \( |1, 0\rangle \) is:

\[
E_{1,0}^{(2)}/\hbar = -\frac{\omega_x^2 + \omega_y^2}{3Q}
\]
One thought on “M05Q.3”

December 13, 2013 at 1:17 am

Everything looks correct.
Notice that you got lucky that the perturbation was diagonal on the degenerate subspace spanned by $|1, 1\rangle$ and $|1, -1\rangle$. Otherwise you would have to first diagonalize the perturbation on this subspace to find the energy correction.