1 May 2005, Quantum, Problem 1

1.1 (a)

$$[\boldsymbol{p}, V] = -i\hbar\nabla V$$

$$< \Psi|[\boldsymbol{p}, H]|\Psi > = < \Psi|\boldsymbol{p}H|\Psi > - \Psi|H\boldsymbol{p}|\Psi > = < \Psi|\boldsymbol{p}|\Psi > E - E < \Psi|\boldsymbol{p}|\Psi > = 0$$

$$< \Psi|[\boldsymbol{p}, H]|\Psi > = < \Psi|[\boldsymbol{p}, V]|\Psi > = -i\hbar < \Psi|\nabla V|\Psi > = 0$$

$$V = -\frac{e^2}{a_B r} + e a_B \boldsymbol{E} \cdot \boldsymbol{r}$$

$$< \Psi|\nabla V|\Psi > = < \Psi|\frac{e^2 \boldsymbol{r}}{a_B^2 r^3}|\Psi > + < \Psi|e\boldsymbol{E}|\Psi > = 0$$

$$\boldsymbol{E'} = < \Psi|\frac{e\boldsymbol{r}}{a_B^2 r^3}|\Psi > = - < \Psi|\boldsymbol{E}|\Psi > = -\boldsymbol{E} \quad (1)$$

1.2 (b)

My idea is to use time-dependent perturbation theory to calculate the final state to first order in E, then compute the expectation value $< \Psi|\frac{\boldsymbol{r}}{r^3}|\Psi >$. The problem (which Silviu pointed out to me) is that the decay rate is said to be large, so that we expect the spontaneous radiation to be much more important than the oscillations. My second idea is to therefore use time-INdependent perturbation theory, but I don't want to take the trouble to do it before I know what's right and what's wrong, so just asking for ideas.