

What should the answer look like? At first glance it appears that this is the standard catenary problem ([Prelim_J00_Mech1](#)). However, there is a subtle but important difference -- here the weight is not distributed along $d\vec{l}$ but dx . This changes the result, but it makes calculations much simpler.

The tensile force is directed along the cable. The horizontal forces are balanced, so parameterize everything in terms of F_H (since it is constant). The tensile force has magnitude $F_H^2 + F_H^2 \tan^2 \phi = F_H \sec \phi$.

The vertical tensile force balances gravity (the span mass length density is $\gamma = \frac{M}{L}$). Working to first order

$$F_H \tan(\phi + d\phi) - F_H \tan \phi - \gamma dx g = 0$$

$$\frac{d \tan \phi}{dx} = \frac{\gamma g}{F_H}$$

$$\sec^2 \phi \frac{d\phi}{dx} = \frac{\gamma g}{F_H}$$

Pleasingly, $\phi = \tan^{-1} \left(\frac{dy}{dx} \right)$ when $dx > 0$, and so

$$\frac{d\phi}{dx} = \frac{1}{\sec^2 \phi} \frac{d^2 y}{dx^2}$$

By the rule for the [Differentiation of Inverse Functions](#). Thus

$$\sec^2 \phi \frac{d\phi}{dx} = \sec^2 \phi \sec^{-2} \phi \frac{d^2 y}{dx^2} = \frac{\gamma g}{F_H}$$

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Yes indeed, suspension cables are more closely parabolic than catenaries.

The equation for y gives

$$y = \frac{\gamma g}{2F_H} x^2 + C_1 x + C_2$$

Let the towers be placed at $x = x_{tower}$ and $x = -x_{tower}$, leaving the origin for the center of the parabola.

Let the height of the towers be $y_{tower} = y(x_{tower}) = y(-x_{tower}) = \frac{\gamma g}{2F_H} x_{tower}^2 + C_2$ (where C_1 is eliminated by symmetry).

Then $C_2 = y_{tower} - \frac{\gamma g}{8F_H} L^2$, where L is the length of the span.

We check to see if F_H is specified by this system by equating forces. Twice the vertical force applied at one of the endpoints is equal to gravitational force on the span, Mg .

The angle at the towers is $\phi_{tower} = \tan^{-1} \left(\frac{dy}{dx} \right) = \tan^{-1} \left(\frac{\gamma L g}{2F_H} \right) = \tan^{-1} \left(\frac{Mg}{2F_H} \right)$.

The horizontal and vertical forces are then $F_H = F_{tot} \cos \phi_{tower}$ and $F_H = \frac{Mg}{2} \cot \phi_{tower}$.

By the formula $\cot \tan^{-1} x = \frac{1}{x}$

$$F_H = \frac{Mg}{2} \frac{2F_H}{Mg} = F_H$$

So we discover that F_H is not specified, any number may be chosen to achieve the desired shape (well, so long as it leads to a cable that's higher than the roadbed).

So, finally, for any reasonable choice of F_H ,

$$y(x) = \frac{Mg}{2F_H L} x^2 + y_{tower} - \frac{Mg}{8F_H} L$$