M04-1 Cycles

a) adiabatic means constant entropy

$$P = -\left(\frac{\partial U}{\partial V}\right)_S$$

$$U = -\int P dV = -\alpha V u(T) + C(S)$$

$$~~V u(T) = \alpha V u(T) + C(S)~~$$

$$\left(\frac{\partial U}{\partial V}\right)\Big|_{S,N} = -P$$

$$u(T) + V\frac{\partial u}{\partial V} = -\alpha u(T)$$

$$u(T) + V\frac{\partial u}{\partial T}\frac{\partial T}{\partial V} = -\alpha u(T)$$

$$V\frac{\partial T}{\partial V}\frac{\partial u}{\partial T} = -(1+\alpha)u$$

$$\frac{\partial u}{\partial T} = -(1+\alpha)\frac{\partial}{\partial T}(\ln V)\, u(T)$$

$$\frac{\partial u}{u} = -(1+\alpha)\,\partial(\ln V)$$

$$\ln u = -(1+\alpha)\ln V + c$$

$$u(T) = C V^{-(1+\alpha)}$$

$$u(T)V^{1+\alpha} = C(S) \quad \text{adiabatic}$$

isothermal has constant temperature ~~PdV = a~~

$$\cancel{dQ = PdV}$$

$$\cancel{TdS = PdV}$$

$\cancel{\delta Q = \alpha u(T) \Delta V}$

$dU = u(T)\, dV$

$dU = dQ - P dV$

$dQ = \alpha u(T) dV + u(T) dV$

$dQ = (\alpha + 1)\, u(T)\, dV$

$\boxed{\Delta Q = (\alpha+1)\, u(T)\, \Delta V}$

$p = \alpha u(T)$

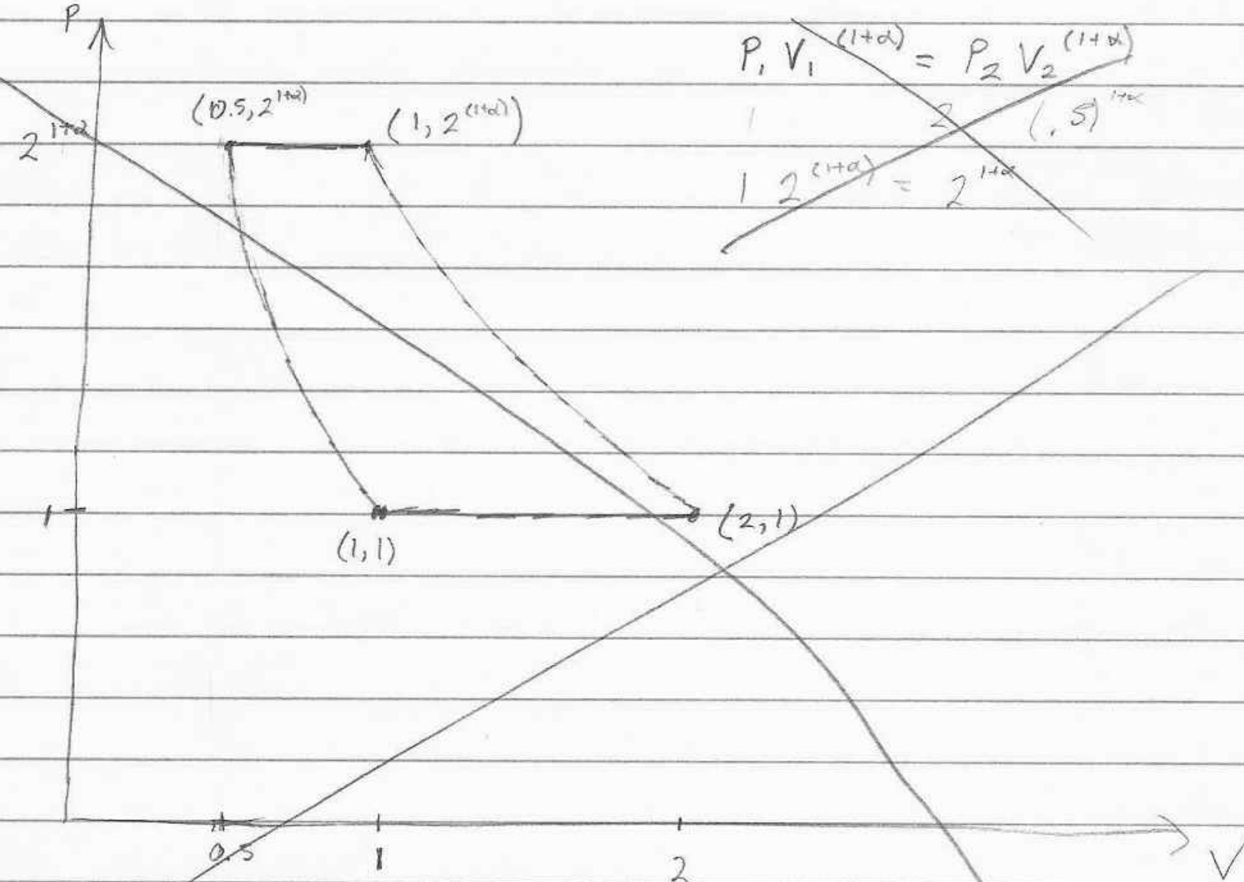
adiabas $p = C(S) V^{-(1+\alpha)}$

$\cancel{P_1 V_1^{(1+\alpha)} = P_2 V_2^{(1+\alpha)}}$

$\cancel{1 \quad 2 \quad (.5)^{1+\alpha}}$

$\cancel{1 \; 2^{(1+\alpha)} = 2^{1+\alpha}}$

b)

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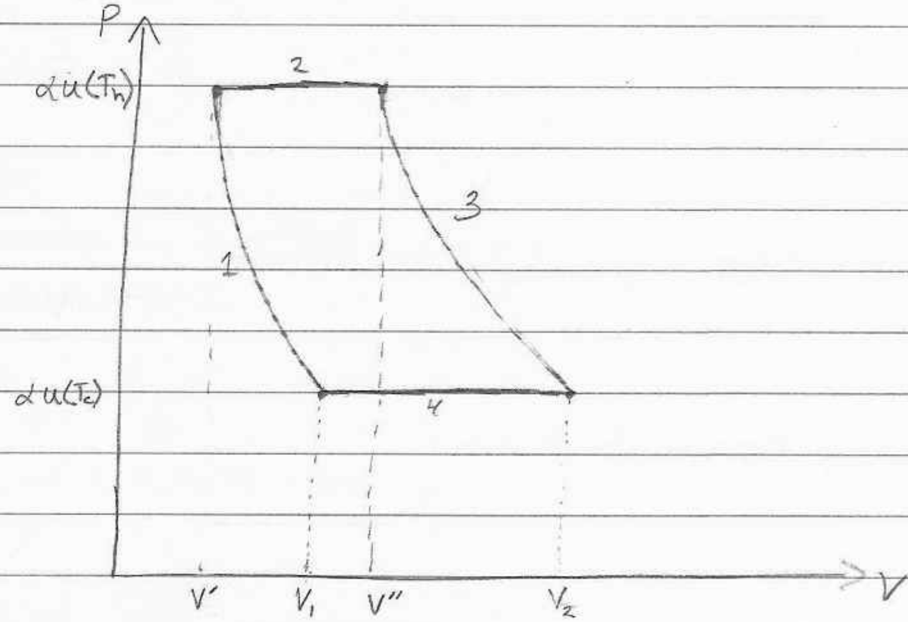
efficiency should be $\frac{W}{Q_h} =$ $P\,dV$

b) generally




well $u(T)\, V^{(1+\alpha)} = \text{const}$

so on 1, $u(T)\, V^{(1+\alpha)} = u(T_c)\, V_1^{(1+\alpha)}$

$$u(T_c)\, V_1^{(1+\alpha)} = u(T_h)\, V'^{(1+\alpha)}$$

$$V' = \left(\frac{u(T_c)}{u(T_h)}\right)^{\frac{1}{1+\alpha}} V_1$$

on 3, $u(T_c)\, V_2^{(1+\alpha)} = u(T_h)\, V''^{(1+\alpha)}$ $\boxed{V'' = \left(\frac{u(T_c)}{u(T_h)}\right)^{1/1+\alpha} V_2}$

on 1: $P = \alpha u(T_c)\, V_1^{(1+\alpha)}\, V^{-(1+\alpha)}$

$$W_1 = + \int_{V'}^{V_1} P\, dV = + \alpha u(T_c)\, V_1^{(1+\alpha)} \left[ -\frac{1}{\alpha} V^{-\alpha} \right]_{V'}^{V_1}$$
$$= -u(T_c)\, V_1^{(1+\alpha)} \left[ V_1^{-\alpha} - \left(\frac{u(T_h)}{u(T_c)}\right)^{\alpha/1+\alpha} V_1^{-\alpha} \right]$$

$$W_1 = -u(T_c)\, V_1 \left[ 1 - \left(\frac{u(T_h)}{u(T_c)}\right)^{\alpha/1+\alpha} \right]$$

$$W_2 = +\alpha\, u(T_h)\,(V''-V') = +\alpha\, u(T_h)\left(\frac{u(T_c)}{u(T_h)}\right)^{1/1+\alpha}(V_2-V_1)$$

on 3, $P = -\alpha\, u(T_c)\, V_2^{(1+\alpha)}\, V^{-(1+\alpha)}$

$$W_3 = +\int_{V''}^{V_2} P\,dV = -u(T_c)V_2\left[1-\left(\frac{u(T_h)}{u(T_c)}\right)^{\alpha/1+\alpha}\right]$$

$$W_4 = +\alpha\, u(T_c)(V_2-V_1)$$

$$W = W_2 + W_3 - W_4 - W_1$$

$$\alpha u(T_h)\left(\frac{u(T_c)}{u(T_h)}\right)^{\frac{1}{1+\alpha}}(V_2-V_1) + u(T_c)V_2\left(\left(\frac{u(T_h)}{u(T_c)}\right)^{\alpha/1+\alpha}-1\right) - \alpha u(T_c)(V_2-V_1)$$
$$- u(T_c)V_1\left(\left(\frac{u(T_h)}{u(T_c)}\right)^{\alpha/1+\alpha}-1\right)$$

$$\alpha(V_2-V_1)\left(\left(\frac{u(T_c)}{u(T_h)}\right)^{1/1+\alpha}u(T_h) - u(T_c)\right) + u(T_c)\left(\left(\frac{u(T_h)}{u(T_c)}\right)^{\alpha/1+\alpha}-1\right)(V_2-V_1)$$

$$u(T_c)(V_2-V_1)\cancel{(\alpha+1)}\left(\left(\frac{u(T_h)}{u(T_c)}\right)^{\frac{\alpha}{1+\alpha}}-1\right) = W$$

$$Q_h = (\alpha+1)\,u(T_h)(V''-V')$$
$$Q_h = (\alpha+1)\,u(T_h)\,(V_2-V_1)\left(\frac{u(T_c)}{u(T_h)}\right)^{1/1+\alpha}$$

$$\mathcal{E} = \frac{W}{Q_h} = \frac{u(T_c)}{u(T_h)}\;\frac{\left(\left(\frac{u(T_h)}{u(T_c)}\right)^{\alpha/1+\alpha}-1\right)}{\left(\frac{u(T_c)}{u(T_h)}\right)^{1/1+\alpha}}$$

$$\mathcal{E} = \frac{u(T_c)}{u(T_h)}\left(\left(\frac{u(T_h)}{u(T_c)}\right) - \left(\frac{u(T_h)}{u(T_c)}\right)^{1/1+\alpha}\right.$$

$$\boxed{\mathcal{E} = 1 - \left(\frac{u(T_c)}{u(T_h)}\right)^{\frac{\alpha}{1+\alpha}}}$$

c) this is the max efficiency & a Carnot cycle has max efficiency

d) $\left[\frac{u(T_c)}{u(T_h)}\right]^{\alpha/1+\alpha} = \frac{T_c}{T_h}$ $\frac{u(T)^{\alpha/1+\alpha}}{T} = \text{const}$

so $\boxed{u(T) \text{ propto } T^{\left(\frac{1+\alpha}{\alpha}\right)}}$

e) photon gas $\boxed{p \text{ ind. of volume}}$