May 2004 QM # 3

a. $f(\theta,\phi) = -\frac{m}{\hbar^2}\hat{V}(\vec{k}-\vec{k}') \qquad \vec{\mathcal{K}} = \vec{k}-\vec{k}'$

$$\hat{V}(\vec{\mathcal{K}}) = \frac{1}{2\pi}\int V(x)\, e^{i\vec{\mathcal{K}}\cdot\vec{x}}\, d^3x$$

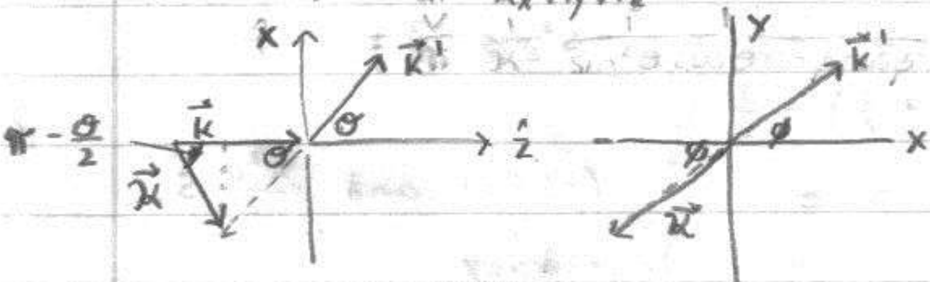
$$= \frac{1}{2\pi}\int_{-L}^{L}\int_{-L}^{L}\int_{-L}^{L} V\, e^{i(\mathcal{K}_x x + \mathcal{K}_y y + \mathcal{K}_z z)}\, dx\, dy\, dz$$

$$= \frac{V}{2\pi}\int_{-L}^{L}\int_{-L}^{L} \frac{1}{i\mathcal{K}_x} e^{i\mathcal{K}_x x}\Big|_{-L}^{L} e^{i\mathcal{K}_y y} e^{i\mathcal{K}_z z}\, dy\, dz$$

$$= \frac{V}{2\pi}\left(\frac{1}{i\mathcal{K}_x}\left(e^{i\mathcal{K}_x L}-e^{-i\mathcal{K}_x L}\right)\right)\left(\frac{1}{i\mathcal{K}_y}\left(e^{i\mathcal{K}_y L}-e^{-i\mathcal{K}_y L}\right)\right)$$

$$\cdot\left(\frac{1}{i\mathcal{K}_z}\left(e^{i\mathcal{K}_z L}-e^{-i\mathcal{K}_z L}\right)\right)$$

$$\hat{V}(\vec{\mathcal{K}}) = \frac{V}{\pi}\frac{1}{\mathcal{K}_x\mathcal{K}_y\mathcal{K}_z}\sin(\mathcal{K}_x L)\sin(\mathcal{K}_y L)\sin(\mathcal{K}_z L)$$



$\mathcal{K}_x = -\mathcal{K}\sin(\pi-\frac{\theta}{2})\cos\phi$

$\to \mathcal{K}_x = -\mathcal{K}\sin(\frac{\theta}{2})\cos\phi$

$\to \mathcal{K}_y = -\mathcal{K}\sin(\frac{\theta}{2})\sin\phi$

$\mathcal{K}_z = -\mathcal{K}\cos(\pi-\frac{\theta}{2})$

$\to \mathcal{K}_z = \mathcal{K}\cos(\frac{\theta}{2})$

$$\mathcal{K} = \sqrt{2k^2 + 2\vec{k}\cdot\vec{k}'}$$

$$\mathcal{K} = k\sqrt{2(1-\cos\theta)} = 2k\sin(\tfrac{\theta}{2})$$

$$\hat{V}(\vec{\mathcal{K}}) = \frac{V}{\pi}\frac{1}{\mathcal{K}^3}\frac{1}{\sin^2(\theta/2)\cos(\phi/2)\sin\phi\cos\phi}\sin(\mathcal{K}\sin(\tfrac{\theta}{2})\cos\phi\, L)$$

$$\cdot\sin(\mathcal{K}\sin(\tfrac{\theta}{2})\sin\phi\, L)\sin(\mathcal{K}\cos(\tfrac{\theta}{2})L)$$

$$\frac{d\sigma}{d\Omega} = |f(\theta,\phi)|^2$$

$$\frac{d\sigma}{d\Omega} = \Big[\frac{m}{\hbar^2}\frac{V}{8k^3}\frac{1}{\sin^5(\theta/2)\cos(\theta/2)\sin\phi\cos\phi}\sin(2k\sin^2(\tfrac{\theta}{2})\cos\phi\, L)$$

$$\cdot\sin(2k\sin^2(\tfrac{\theta}{2})\sin\phi\, L)\sin(2k\sin(\tfrac{\theta}{2})\cos(\tfrac{\theta}{2})L)\Big]^2$$

b. This approximation is vaid if the potential is "small". More specifically, the amplitude of the scattered wave must be small compared to the amplitude of the incident wave. This allows us to use $e^{i\vec{k}\cdot\vec{x}}$ for $\psi(x)$ in the born integral and neglect the rest of the wavefunction which would produce second and higher order terms)