

We have the **Larmor formula** for the total power radiated by an accelerating charge:

$$P_{rad} = \frac{2e^2 a^2}{3c^3}$$

The force doing the accelerating is:

$$F_{coulomb} = ma = -\frac{e^2}{r^2} \Rightarrow a = -\frac{e^2}{mr^2}$$

So that:

$$P_{rad} = \frac{2e^2}{3c^3} \left( -\frac{e^2}{mr^2} \right)^2 = \frac{2e^6}{3c^3 m^2 r^4}$$

The total energy is:

$$E = \frac{1}{2}mv^2 + \int P_{rad} dt - \frac{e^2}{r}$$

We find the velocity as a function of radius, since it has a nearly circular orbit, by:

$$F_{centrip} = F_{coulomb} \Rightarrow \frac{mv^2}{r} = \frac{e^2}{r^2} \Rightarrow v = \sqrt{\frac{e^2}{mr}}$$

So we can write the total energy:

$$E_0 = -\frac{e^2}{2r} + \int \frac{2e^6}{3c^3 m^2 r^4} dt$$

Differentiating with respect to time:

$$0 = \frac{e^2}{2r^2} \frac{dr}{dt} + \frac{2e^6}{3c^3 m^2 r^4}$$

Rearranging:

$$-\frac{3c^3 m^2 r^2}{4e^4} \frac{dr}{dt} = 1$$

Integrating with respect to time:

$$const - \frac{c^3 m^2 r^3}{4e^4} = t$$

Since at  $t=0$ ,  $r=r_i$ :

$$t = \frac{c^3 m^2}{4e^4} (r_i^3 - r^3) \approx .1 \text{ ns}$$