M03T.1

a) We utilise the free energy $dF = -SdT + fdL$. From this and $f = \mu x - \alpha T + \beta T x$

$$\left.\frac{\partial S}{\partial x}\right|_T = -\left.\frac{\partial f}{\partial T}\right|_x = \alpha - \beta x$$

b)
To verify this we examine the derivative of C_x with respect to x

$$\left.\frac{\partial C_x}{\partial x}\right|_T = \left.\frac{\partial}{\partial x}\right|_T T \left.\frac{\partial S}{\partial T}\right|_x = T \left.\frac{\partial}{\partial T}\right|_x \left.\frac{\partial S}{\partial x}\right|_T = 0$$

Here we have made use of the result from the previous part and the fact that the derivates can be taken in either order.

c)
The first part is easily attained from the heat capacity:

$$\left.\frac{\partial S}{\partial T}\right|_x = \frac{C_x}{T} = A \tag{1}$$

For the next part we integrate the previous answer and the answer from part a.

$$S = AT + f(x)$$
$$S = (\alpha - \beta \frac{x}{2})x + g(T)$$

From this, and the condition that $S(0,0) = B$, we conclude that $S = AT + (\alpha - \beta \frac{x}{2})x + B$

d)

The condition $f = 0$ allows us to calculate $\frac{\partial x}{\partial T}\Big|_{f=0} = \frac{\mu\alpha}{(\mu+\alpha T)^2}$. Using this we immediately see:

$$C_F = T\,\frac{\partial S}{\partial T}\Big|_{f=0} = AT + \frac{\alpha^2 T\mu}{(\mu+\beta T)^2} - \frac{\beta\mu\alpha^2 T^2}{(\mu+\beta T)^3} = AT + \frac{\alpha^2 T\mu^2}{(\mu+\beta T)^3}$$

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2 thoughts on “M03T.1”

December 16, 2013 at 1:46 pm

Better now. You've lost a factor of T in part (d) and have some typos there. Fix that and you will be done with this problem.

Mykola Dedushenko
December 15, 2013 at 11:15 pm

In (b) the idea is right, but the implementation is not. $\delta Q = TdS$ is not a full differential in general (i.e. there is no a well defined function Q to differentiate), so it is NOT correct to write $C_x = \left(\frac{\partial Q}{\partial T}\right)_x$. Rather $C_x = T\left(\frac{\partial S}{\partial T}\right)_x$. If you use this, you'll get the correct derivation of (b).
In part (c) again the idea is right but the formulae are not for no obvious reason. Just redo this part -- you have $\partial S/\partial T$ and $\partial S/\partial x$ from what you've done before, just use it accurately.
Part (d) is dependent on your answer in (c) so your have to redo it too.
