

Taking the time derivative of the rolling without slipping condition:

$$\frac{d^2 \vec{R}}{dt^2} = a \frac{d\vec{\omega}}{dt} \times \hat{n}$$

The only torque on the ball is the force to enforce rolling without friction:

$$I \dot{\vec{\omega}} = a \hat{r} \times \vec{f}$$

Rearranging:

$$\frac{d\vec{\omega}}{dt} = \frac{a}{I} \hat{r} \times \vec{f}$$

Plugging this in:

$$\frac{d^2 \vec{R}}{dt^2} = \frac{a^2}{I} \hat{r} \times \vec{f} \times \hat{n}$$

Since \hat{r} , so that $\hat{r} \times \vec{f} \times \hat{n} = -\vec{f}$:

$$\vec{f} = -\frac{I}{a^2} \frac{d^2 \vec{R}}{dt^2}$$

Writing the force equation in the downhill direction:

$$m \frac{d^2 R}{dt^2} = mg \sin \theta - \frac{I}{a^2} \frac{d^2 R}{dt^2}$$

For a sphere $I = \frac{2}{5} ma^2$:

$$m \frac{d^2 R}{dt^2} = mg \sin \theta - \frac{2}{5} m \frac{d^2 R}{dt^2} \quad \frac{7}{5} \frac{d^2 R}{dt^2} = g \sin \theta$$

So we finally get:

$$\frac{d^2 R}{dt^2} = \frac{5}{7} g \sin \theta$$

We change the condition for rolling without slipping to be:

$$\frac{d\vec{R}}{dt} + \vec{\Omega} \times \vec{R} = a\vec{\omega} \times \hat{n}$$

to take into account the motion of the ground. Taking the time derivative:

$$\frac{d^2 \vec{R}}{dt^2} + \vec{\Omega} \times \frac{d\vec{R}}{dt} = a \frac{d\vec{\omega}}{dt} \times \hat{n}$$

\vec{f} will be defined in the same way as before:

$$\frac{d^2 \vec{R}}{dt^2} + \left(\vec{\Omega} \times \frac{d\vec{R}}{dt} \right) = -\frac{a^2}{I} \vec{f}$$

So that in the force equation:

$$m \frac{d^2 \vec{R}}{dt^2} = -\frac{2}{5} m \frac{d^2 \vec{R}}{dt^2} + -\frac{2}{5} m \left(\vec{\Omega} \times \frac{d\vec{R}}{dt} \right)$$

or:

$$\frac{7}{5} \frac{d^2 \vec{R}}{dt^2} = -\frac{2}{5} \left(\vec{\Omega} \times \frac{d\vec{R}}{dt} \right)$$

Since $\vec{\Omega}$ is perpendicular to the plane:

$$\frac{d^2 \vec{R}}{dt^2} = -\frac{2}{7} \Omega \left(\hat{z} \times \frac{d\vec{R}}{dt} \right)$$

This is the equation of motion for a circle in the plane, moving with frequency $\frac{2}{7} \Omega$!