In the rotating frame, the gravity must provide the centripetal force:

\[ F_c = m r \omega^2 \]

The force of gravity is:

\[ F_g = \frac{GM_s m}{r^2} + \frac{GM_e m}{(r - R)^2} \]

For equilibrium:

\[ m r \omega^2 = \frac{GM_s m}{r^2} + \frac{GM_e m}{(r - R)^2} \]

The left hand side of this equation is monotonically increasing, while for \( r > R \), the RHS is monotonically decreasing from \( \infty \) to 0. Therefore, there is one solution to this equation for \( r > R \).

We can rearrange the equation:

\[ r^3 \omega^2 = GM_s \left( 1 + \beta \frac{r^2}{(r - R)^2} \right) \]

Taking the cube root of both sides, and expanding the cube root of what is in parenthesis in terms of small parameter \( \beta \):

\[ r = \left( \frac{GM_s}{\omega^2} \right)^{1/3} \left( 1 + \beta \frac{r^2}{3 (r - R)^2} \right) \]

We can make the replacement \( x = r - R \), and abbreviate \( k = \left( \frac{GM_s}{\omega^2} \right)^{1/3} \) to find the distance:

\[ x + R = \left( \frac{GM_s}{\omega^2} \right)^{1/3} \left( 1 + \beta \frac{(x + R)^2}{x^2} \right) \]

Putting this in order, the distance \( x \) from the earth solves:

\[ x^3 + \left( R + \frac{\beta k}{3} - k \right) x^2 + \frac{2 \beta k}{3} R x + \frac{\beta k}{3} R^2 = 0 \]

The force equation is:

\[ m \ddot{r} = m r \omega^2 - \frac{GM_s m}{r^2} - \frac{GM_e m}{(r - R)^2} \]

Perturbing around a solution \( r_{L2} \), so that \( r = r_{L2} + \delta r \):

\[ m \ddot{r} = m \omega^2 \delta r + \frac{GM_s m}{r_{L2}} \delta r + \frac{GM_e m}{r_{L2} - R} \delta r = \left( m \omega^2 + \frac{GM_s m}{r_{L2}} + \frac{GM_e m}{r_{L2} - R} \right) \delta r \]

\( \dot{r} = \left( \omega^2 + \frac{GM_s}{r_{L2}} + \frac{GM_e}{r_{L2} - R} \right) \delta r \)

This coefficient is always positive (since \( r_{L2} \) is defined to be outside earth's orbit), so the solution is unstable.