In the rotating frame, the gravity must provide the centripetal force:

$$F_c = mr\omega^2$$

The force of gravity is:

$$F_g = \frac{GM_s m}{r^2} + \frac{GM_e m}{(r-R)^2}$$

For equilibrium:

$$mr\omega^2 = \frac{GM_s m}{r^2} + \frac{GM_e m}{(r-R)^2}$$

The left hand side of this equation is monotonically increasing, while for r>R , the RHS is monotonically decreasing from ∞ to 0. Therefore, there is one solution to this equation for r>R .

We can rearrange the equation:

$$r^3\omega^2 = GM_s\left(1 + \beta\frac{r^2}{(r-R)^2}\right)$$

Taking the cube root of both sides, and expanding the cube root of what is in parenthesis in terms of small parameter β:

$$r = \left(\frac{GM_s}{\omega^2}\right)^{1/3}\left(1 + \frac{\beta}{3}\frac{r^2}{(r-R)^2}\right)$$

We can make the replacement x=r-R , and abbreviate $k = \left(GM_s/\omega^2\right)^{1/3}$ to find the distance:

$$x + R = \left(\frac{GM_s}{\omega^2}\right)^{1/3}\left(1 + \frac{\beta}{3}\frac{(x+R)^2}{x^2}\right)$$

Putting this in order, the distance x from the earth solves:

$$x^3 + \left(R + \frac{\beta k}{3} - k\right)x^2 + \frac{2\beta k}{3}Rx + \frac{\beta k}{3}R^2 = 0$$

The force equation is:

$$m\ddot{r} = mr\omega^2 - \frac{GM_s m}{r^2} - \frac{GM_e m}{(r-R)^2}$$

Perturbing around a solution r_{L2}, so that $r = r_{L2} + \delta r$:

$$m\ddot{\delta r} = m\omega^2\delta r + \frac{GM_s m}{r_{L2}}\delta r + \frac{GM_e m}{r_{L2}-R}\delta r = \left(m\omega^2 + \frac{GM_s m}{r_{L2}} + \frac{GM_e m}{r_{L2}-R}\right)\delta r$$

$$\ddot{\delta r} = \left(\omega^2 + \frac{GM_s}{r_{L2}} + \frac{GM_e}{r_{L2}-R}\right)\delta r$$

This coefficient is always positive (since r_{L2} is defined to be outside earth's orbit), so the solution is unstable.