M03E.1

Draft solution. Consider cylindrical coordinates and $\mathbf{E}$ and $\mathbf{B}$ fields at a point $(\rho, \phi, z) = (r, 0, 0)$ above a wire. Since the wire is neutral, $V = 0$ and $\mathbf{E} = 0$ in each case.

1. $$\mathbf{A}(r,0,0,t) = \frac{\mu_0}{4\pi} \int \frac{I(z, t - \mathfrak{r}/c)}{\mathfrak{r}}\, d\mathbf{z}$$

with $\mathfrak{r} = \sqrt{z^2 + r^2}$.

$$A_z = \frac{\mu_0}{4\pi} \int_{-z_0}^{z_0} \frac{I_0}{\sqrt{z^2 + r^2}}\, dz$$

where $z_0 = \sqrt{c^2 t^2 - r^2}$. Integrating,

$$A_z = \frac{\mu_0 I_0}{4\pi} \ln\left[\frac{ct + \sqrt{c^2 t^2 - r^2}}{ct - \sqrt{c^2 t^2 - r^2}}\right].$$

Using $\mathbf{B} = \nabla \times \mathbf{A}$,

$$B_\phi = -\frac{\partial A_z}{\partial r} = \frac{\mu_0 I_0 ct}{2\pi r \sqrt{c^2 t^2 - r^2}}.$$

The force per unit length may then be obtained as

$$F_l = B_\phi I_0 = \frac{\mu_0 I_0^2 ct}{2\pi r \sqrt{c^2 t^2 - r^2}}.$$

Singular at $r = ct$ -- physically unrealistic.

2. We divide the wire into segment $[-z_0, z_0]$ where the current is increasing linearly and two segments $[\pm z_0, \pm z_1]$ where the current has it's maximum value.

$$A_z = \frac{\mu_0}{4\pi}\int_{-z_0}^{z_0} dz \frac{I_0}{\sqrt{z^2+r^2}} + 2\,\frac{\mu_0 b}{4\pi}\int_{z_0}^{z_1} dz \left[\frac{t}{\sqrt{z^2+r^2}} - \frac{1}{c}\right]$$

where $z_0 = \sqrt{c^2(t-(I_0/b))^2 - r^2}$ and $z_1 = \sqrt{c^2t^2 - r^2}$. The $\mathbf{B}$-field and force per unit length may then be obtained as before.

$$B_\phi = -\frac{\partial A_z}{\partial r}$$

$$F_l = B_\phi I_0$$