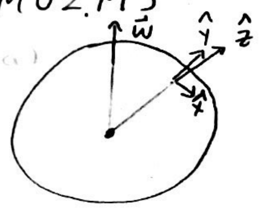
M02.M3



a) $m\ddot{z} = -mg + k\dot{z}^2$

$$\frac{d\dot{z}}{dt} = -g + \frac{k}{m}\dot{z}^2$$

$$\int \frac{d\dot{z}}{-g\left(1-\frac{k}{mg}\dot{z}^2\right)} = \int dt \quad \text{where } \left|\sqrt{\frac{k}{mg}}\,\dot{z}\right| < 1$$

$$-\frac{1}{g}\sqrt{\frac{mg}{k}}\tanh^{-1}\left(\sqrt{\frac{k}{mg}}\,\dot{z}\right) = t + C$$

$$\tanh\left(-\sqrt{\frac{kg}{m}}\,t + C\right) = \sqrt{\frac{k}{mg}}\,\dot{z}$$

$$\dot{z} = -\sqrt{\frac{mg}{k}}\tanh\left(\sqrt{\frac{kg}{m}}t + C\right)$$

$$\dot{z}(t=0) = 0 \Rightarrow C = 0$$

$$\boxed{\dot{z} = -\sqrt{\frac{mg}{k}}\tanh\left(\sqrt{\frac{kg}{m}}\,t\right)}$$

b) Coriolis acceleration:

$$\vec{a}_c = -2\vec{w} \times \dot{z}\hat{z} = \ddot{y}\hat{y}$$

$$\vec{w} = (-w\cos\lambda,\ 0,\ w\sin\lambda)$$

$$\vec{\dot{z}} = (0,\ 0,\ \dot{z})$$

$$\vec{w} \times \vec{\dot{z}} = w\dot{z}\cos\lambda\,\hat{y}$$

$$\ddot{y} = -2w\dot{z}\cos\lambda$$

$$\ddot{y} = +2w\cos\lambda\sqrt{\frac{mg}{k}}\tanh\left(\sqrt{\frac{kg}{m}}\,t\right)$$

$$= 2w\cos\lambda\sqrt{\frac{mg}{k}}\,\frac{\sinh(\sqrt{kg/m}\,t)}{\cosh(\sqrt{kg/m}\,t)}$$

$$\dot{y} = 2w\cos\lambda\sqrt{\frac{mg}{k}}\sqrt{\frac{m}{kg}}\log\left(\cosh\left(\sqrt{\frac{kg}{m}}\,t\right)\right) + C$$

$$\dot{y} = 2w\cos\lambda\,\frac{m}{k}\log\left(\cosh\left(\sqrt{\frac{kg}{m}}\,t\right)\right) + C$$

$$\dot{y}(0) = 0 \Rightarrow C = 0$$

$$\boxed{\dot{y} = 2w\cos\lambda\,\frac{m}{k}\log\left(\cosh\left(\sqrt{\frac{kg}{m}}\,t\right)\right)}$$

c) Let $t^* = n\sqrt{\frac{m}{kg}}$, $n \in \mathbb{Z}^+$

$\dot{y}$:

$$\lim_{t^*\to\infty}\dot{y}(t^*) = \lim_{n\to\infty}\dot{y}(t^*(n))$$

When n is large,

$$\log(\cosh(n)) \approx n$$

$$\lim_{t^*\to\infty}\dot{y}(t^*) \longrightarrow 2\cos\lambda\,\frac{mw}{k}\sqrt{\frac{kg}{m}}\,t^*$$

$$\boxed{\dot{y} \longrightarrow 2wt\cos\lambda\sqrt{\frac{mg}{k}}}$$

$\dot{z}$

$$\lim_{t^*\to\infty}\dot{z}(t) = -\sqrt{\frac{mg}{k}}$$

$$\boxed{\dot{z} \longrightarrow -\sqrt{\frac{mg}{k}}}$$